This question paper contains **4+1** printed pages]

BF—66—2016

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (First Year) (First Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

(CBCS Pattern)

MATHEMATICS

Paper I

(Differential Calculus)

(MCQ + Theory)

(Wednesday, 19-10-2016) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :—(*i*) Attempt *All* questions.

(*ii*) *One* mark to each correctly answered MCQ.

(*iii*) Negative marking system is applicable.

(*iv*) Use black ball point pen to darken the circle of correct choice in OMR answer-sheet. Circle once darkened is final. No change is permitted.

(*v*) Darken only one circle for the answer of an MCQ.

MCQ

1. Choose the correct alternative for each of the following : 1 each

(1) The derivative of sech x, $x \in R$ is :

(*a*) tanh x. sech x (*b*) –tanh x. sech x

(*c*) coth x. sech x (*d*) –coth x. sech x

P.T.O.

(2) $\frac{d^n}{dx^n}[(ax+b)^{-1}]$ = ...

(a) $\frac{(-1)^n . n!\, a^n}{(ax+b)^{n+1}}$ (b) $\frac{(-1)^n . (n+1)!\, a^n}{(ax+b)^{n+1}}$

(c) $\frac{(-1)^n . n!\, a^n}{(ax+b)^{n}}$ (d) $\frac{(-1)^n . n!\, a^{n+1}}{(ax+b)^{n+1}}$

(3) The equation of the tangent at a point (x, y) on the curve $y = f(x)$ is :

(a) $X - x = f'(x)\ (Y - y)$

(b) $X - x = -f'(x)\ (Y - y)$

(c) $Y - y = f'(x)\ (X - x)$

(d) $Y - y = -f'(x)\ (X - x)$

(4) The length of the subnormal at any point of the curve $y = f(x)$ is :

(a) $y.\frac{dy}{dx}$ (b) $y.\frac{dx}{dy}$

(c) $y.\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ (d) $y.\sqrt{1+\left(\frac{dx}{dy}\right)^2}$

(5) If a function f is :

(i) continuous in $[a, b]$

(ii) derivable in $]a, b[$

(iii) $f(a) = f(b)$

then there exists at least one value $c \in\ ]a, b[$ such that :

(a) $f'(c) < 0$ (b) $f'(c) \neq 0$

(c) $f'(c) = 0$ (d) $f'(c) > 0$

(6) Cauchy form of remainder after n terms in Taylor's theorem is :

(*a*) $\frac{h^n.(1-\theta)^{n-p}}{p(n-1)!} f^n(a + \theta h)$

(*b*) $\frac{h^{n-1}(1-\theta)^{n-1}}{(n-1)!} f^n(a + \theta h)$

(*c*) $\frac{h^n}{n!} f^n(a + \theta h)$

(*d*) $\frac{h^n}{(n-1)!} f^n(a + \theta h)$

(7) $\lim_{x \to 0} x. \log x =$ ....

(*a*) 0 (*b*) 1

(*c*) –1 (*d*) ∞

(8) $\lim_{k \to 0} \frac{f(a, b+k) - f(a, b)}{k}$, if it exists, is called the partial derivative of $f(x, y)$ w.r.t. y at (a, b) and is denoted by :

(*a*) $f_{xy}(a, b)$ (*b*) $f_{yx}(a, b)$

(*c*) $f_x(a, b)$ (*d*) $f_y(a, b)$

(9) If $z = e^{ax}. \sin by$, then $\frac{\partial z}{\partial x}$ = ...

(*a*) $ae^{ax}. \cos by$ (*b*) $ae^{ax}. \sin by$

(*c*) $be^{ax} \cos by$ (*d*) $be^{ax} \sin by$

(10) $\frac{\partial}{\partial y}\left(e^{xyz}\right)$ = ...

(*a*) $xyz \, . \, e^{xyz}$ (*b*) $xy \, e^{xyz}$

(*c*) $xz \, e^{xyz}$ (*d*) $yz \, e^{xyz}$

Theory

2. Attempt any *two* of the following : 5 each

(*a*) Prove that :

$$\frac{d^n}{dx^n}[e^{ax}. \cos(bx + c) = r^n . e^{ax}. \cos(bx + c + n\phi)]$$

where $r = \sqrt{a^2 + b^2}$, $\phi = \tan^{-1}\left(\frac{b}{a}\right)$.

(*b*) If u and v are two functions of x possessing derivatives of the nth order, then prove that :

$$(uv)_n = u_n v + nc_1 u_{n-1} v_1 + nc_2 u_{n-2} + ... + nc_r u_{n-r} v_r + ... + nc_n u.v_n.$$

(*c*) Show that the length of the portion of the tangent to the curve

$$x = a \cos^3 \theta,$$

$$y = a \sin^3 \theta$$

intercepted between the coordinate axes is constant.

3. Attempt any *two* of the following : 5 each

(*a*) If a function f is (*i*) continuous in a closed interval $[a, b]$ and (*ii*) derivable in the open interval $]a, b[$, then prove that there exists at least one value $c \in]a, b[$ such that :

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

(*b*) If in the Cauchy mean value theorem, $f(x) = e^x$ and $F(x) = e^{-x}$, then show that c is arithmetic mean between a and b.

(*c*) Determine :

$$\lim\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right).$$

4. Attempt any *two* of the following : 5 each

(*a*) If

$$z = f(x, y)$$

be a homogeneous function of x, y of degree n, then prove that :

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz.$$

(*b*) If

$$u = \frac{1}{\sqrt{x^2 + y^2 + z^2}},\ x^2 + y^2 + z^2 \neq 0$$

then show that :

$$\frac{\partial^2 u}{dx^2} + \frac{\partial^2 u}{dy^2} + \frac{\partial^2 u}{dz^2} = 0.$$

(*c*) If

$$u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right),\ x \neq y,$$

then show that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$$