This question paper contains **4+1** printed pages]

BF—67—2016

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (First Year) (First Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

(Old Course)

MATHEMATICS

Paper I

(Differential Calculus)

(MCQ + Theory)

(Wednesday, 19-10-2016) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—10+30=40

N.B. :—(*i*) Attempt *All* questions.

(*ii*) One mark to each correctly answered MCQ.

(*iii*) Negative marking system is applicable.

(*iv*) Use black ball point pen to darken the circle of correct choice in OMR answer-sheet.

(*v*) Circle once darkened is final. No change is permitted.

(*vi*) Darken only one circle for the answer of an MCQ.

MCQ

1. Choose the *correct* alternative for each of the following :

(1) The derivative of tanh x, $x \in$ R is :

(*a*) sech x. tanh x

(*b*) –sech x. tanh x

(*c*) –sech^2 x

(*d*) sech^2 x

P.T.O.

(2) $\dfrac{d^n}{dx^n}\left(\dfrac{1}{ax+b}\right) =$ ....

(a) $\dfrac{(-1)^n (n!) a^n}{(ax + b)^{n+1}}$ (b) $\dfrac{(-1)^n (n!) a^n}{(ax + b)^{n-1}}$

(c) $\dfrac{(-1)^n n! a^n}{(ax + b)^n}$ (d) $\dfrac{(-1)^n n! a^{n+1}}{(ax + b)^{n+1}}$

(3) The equation of tangent at the point 't' of the curve $x = f(t)$, $y = F(t)$ is :

(a) $[X - f(t)]\ F'(t) - [Y - F(t)]\ f'(t) = 0$

(b) $[X - f(t)]\ F(t) - [Y - F(t)]\ f'(t) = 0$

(c) $[X + f(t)]\ F'(t) - [Y + F(t)]\ f'(t) = 0$

(d) $[X - F'(t)]\ f(t) - [Y - F(t)]\ f'(t) = 0$

(4) The length of the subnormal at any point of the curve $y = f(x)$ is :

(a) $y\dfrac{dx}{dy}$ (b) $y\dfrac{dy}{dx}$

(c) $x\sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}$ (d) $\sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}$

(5) If a function f is :

(i) continuous in a closed interval $[a, b]$

(ii) derivable in the open interval $]a, b[$

(iii) $f(a) = f(b)$.

Then there exists at least one value 'c' $\in\ ]a, b[$ such that $f'(c) = 0$. This statement is known as :

(a) Lagrange's mean value theorem

(b) Rolle's theorem

(c) Cauchy's mean value theorem

(d) Taylor's theorem

(6) Cauchy's form of remainder in Maclaurin's series is :

(*a*) $\dfrac{x^n (1-\theta)^{n-p} f^n(\theta x)}{p(n-1)!}$

(*b*) $\dfrac{x^n (1-\theta)^{n-1}}{(n-1)!} f^n(\theta x)$

(*c*) $\dfrac{x^n}{n!} f^n(\theta x)$

(*d*) None of the above

(7) The value of $\lim\limits_{x \to 0} \dfrac{1-\cos x}{3x^2}$ is :

(*a*) 3 (*b*) $\dfrac{1}{3}$

(*c*) $\dfrac{1}{6}$ (*d*) $\dfrac{1}{9}$

(8) $\lim\limits_{k \to 0} \dfrac{f(x, y+k) - f(x, y)}{k}$ if it exists is called partial derivative of f w.r.t. y at (x, y) is denoted by :

(*a*) $f_x(x, y)$ (*b*) $f_{yx}(x, y)$

(*c*) $f_{xy}(x, y)$ (*d*) $f_y(x, y)$

(9) If $z = e^{ax} \sin by$, then $\dfrac{\partial z}{\partial x}$ = ...

(*a*) $ae^{ax} \sin by$ (*b*) $ae^{ax} \cos by$

(*c*) $-ae^{ax} \cos by$ (*d*) $-ae^{ax} \sin by$

(10) If $u = \log(\tan x + \tan y + \tan z)$, then $\frac{\partial u}{\partial x}$ = ...

(*a*) $\frac{-\sec^2 x}{\tan x + \tan y + \tan z}$

(*b*) $\frac{\sec^2 x}{\tan x + \tan y + \tan z}$

(*c*) $\frac{1}{\tan x + \tan y + \tan z}$

(*d*) $\frac{-1}{\tan x + \tan y + \tan z}$

2. Attempt any *two* of the following : 5 each

(*a*) Prove that :

$$\frac{d^n}{dx^n} \sin(ax + b) = a^n \sin\left(ax + b + \frac{n\pi}{2}\right).$$

(*b*) If u and v be two functions of x possessing derivatives of nth order, then prove that :

$$(uv)_n = u_n v + nc_1\, u_{n-1}\, v_1 + nc_2\, u_{n-2}\, v_2 + ... + nc_n\, uv_n$$

(*c*) Find the angle of intersection of the parabolas

$$y^2 = 4ax \text{ and}$$

$$x^2 = 4by$$

at their point of intersection other than the origin.

3. Attempt any *two* of the following : 5 each

(*a*) If two functions $f(x)$ and $F(x)$ are derivable in closed interval $[a, b]$ and $F'(x) \neq 0$ for any value of x in $[a, b]$, then prove that there exists at least one value 'c' of x belonging to the open interval $]a, b[$ such that :

$$\frac{f(b)-f(a)}{F(b)-F(a)}=\frac{f'(c)}{F'(c)}.$$

(*b*) Verify Rolle's theorem for the function :

$$f(x) = (x - a)^m (x - b)^n,$$

where m, n being positive integers and $x \in [a, b]$.

(*c*) Find :

$$\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x}}.$$

4. Attempt any *two* of the following : 5 each

(*a*) If $f(x, y)$ possesses continuous second order partial derivatives f_{xy} and f_{yx}, then prove that $f_{xy} = f_{yx}$.

(*b*) If

$$u = 3(lx + my + nz)^2 - (x^2 + y^2 + z^2) \text{ and}$$
$$l^2 + m^2 + n^2 = 1$$

then show that :

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0.$$

(*c*) If

$$z = (x + y)\,\phi(y/x)$$

where ϕ is any arbitrary function, then prove that :

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z.$$