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BF—81—2016

FACULTIES OF ARTS/SCIENCE

B.A./B.Sc. (First Year) (First Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

MATHEMATICS

Paper II (CBCS Pattern)

(Algebra and Trigonometry)

(MCQ+Theory)

(Friday, 21-10-2016) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) First **30** minutes for Question No. **1** (MCQ) and remaining time for theory.

(*iii*) Negative marking system is applicable for MCQ.

(*iv*) Use black ball point pen to darken the circle of correct answer in OMR answer-sheet. Circle once darkened is final.

(*v*) Figures to the right indicate full marks.

(MCQ)

1. Choose the most correct alternative for each of the following : 1 each

(*i*) If A and B are confirmable matrices for multiplication, then which of the following statements is correct ?

(*a*) $(AB)' = A' . B'$

(*b*) $(\overline{AB}) = \overline{A} . \overline{B}$

(*c*) $(AB)^{-1} = A^{-1} . B^{-1}$

(*d*) $(AB)^{\theta} = A^{\theta} . B^{\theta}$

(*ii*) A square matrix A is said to be idempotent matrix if ....

(*a*) $A^2 = A$ (*b*) $A^2 = I$

(*c*) $A^2 = 0$ (*d*) $|A| = 0$

P.T.O.

(*iii*) If :

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

and C_{ij} denote co-factor of a_{ij} in det (A), then $a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13}$ = ...

(*a*) zero (*b*) one

(*c*) det (A) (*d*) adj (A)

(*iv*) For a non-singular matrix A, A^{-1} = ...

(*a*) $|A|$ adj (A) (*b*) adj (A) $|A|$

(*c*) $\dfrac{\text{adj (A)}}{|A|}$ (*d*) $\dfrac{|A|}{\text{adj (A)}}$

(*v*) The rank of the matrix $\begin{bmatrix} 1 & -2 & -1 & 4 \\ 2 & -4 & 3 & 5 \\ -1 & 2 & 6 & -7 \end{bmatrix}$ is :

(*a*) 3 (*b*) 1

(*c*) 0 (*d*) 2

(*vi*) The set of characteristic roots of the matrix A is called ....

(*a*) spectrum of matrix A (*b*) pole of matrix A

(*c*) latent of matrix A (*d*) eigenvalue of matrix A

(*vii*) If A is a matrix or order $m \times n$, then :

(*a*) $\rho(A) \le \max.(m, n)$ (*b*) $\rho(A) \le \min(m, n)$

(*c*) $\rho(A) > \max(m, n)$ (*d*) $\rho(A) > \min(m, n)$

(*viii*) If x is real or complex, then $\sin x$ = ...

(*a*) $\dfrac{e^{x} - e^{-x}}{2}$ (*b*) $\dfrac{e^{ix} - e^{-ix}}{2i}$

(*c*) $\dfrac{e^{ix} + e^{-ix}}{2}$ (*d*) $\dfrac{e^{ix} - e^{-ix}}{2}$

(*ix*) The value of :

$$\frac{(\cos 2\theta - i \sin 2\theta)^7 (\cos 3\theta + i \sin 3\theta)^{-5}}{(\cos 4\theta + i \sin 4\theta)^{12} (\cos 5\theta - i \sin 5\theta)^{-6}}$$

is :

(*a*) $\cos 109\theta + i \sin 109\theta$

(*b*) $\cos 100\theta + i \sin 100\theta$

(*c*) $\cos 109\theta - i \sin 109\theta$

(*d*) $\cos 107\theta - i \sin 107\theta$

(*x*) The value of $\sinh^{-1} x$ is :

(*a*) $\log(x + \sqrt{x^2 + 1})$ (*b*) $\log(x + \sqrt{x^2 - 1})$

(*c*) $\frac{1}{2}\log\left|\frac{1 + x}{1 - x}\right|$ (*d*) $\frac{1}{2}\log\left|\frac{x - 1}{x + 1}\right|$

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) If A, B, C are matrices of the order $m \times n$, $n \times p$, $p \times q$ respectively, then prove that :

$$(AB)C = A(BC).$$

(*b*) Prove that the necessary and sufficient condition for a square matrix A to possess the inverse is that :

$$|A| \neq 0$$

(i.e., A is non-singular).

(*c*) Find the inverse of the matrix A given by :

$$A = \begin{bmatrix} 9 & 5 & 6 \\ 7 & -1 & 8 \\ 3 & 4 & 2 \end{bmatrix}.$$

3. Attempt any *two* of the following : 5 each

(*a*) If AX = 0 is a homogeneous system of equations in n unknowns and $X_1 = (x_1, x_2 \ldots\ldots\ldots x_n)$ and $X_2 = (y_1, y_2 \ldots\ldots\ldots y_n)$ are two solutions of this system, then $X_1 + X_2 = (x_1 + y_1, x_2 + y_2, \ldots\ldots\ldots, x_n + y_n)$ is also a solution. Also, if λ is a scalar, then $\lambda X_i = (\lambda x_1, \lambda x_2, \ldots\ldots \lambda x_n)$ is also a solution.

(*b*) Prove that λ is a characteristic root of a matrix A if and only if there exists a non-zero vector X such that $AX = \lambda X$.

(*c*) Examine the consistency of the following equations and if consistent, find the complete solution :

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$x + 4y + 7z = 30.$$

4. Attempt any *two* of the following : 5 each

(*a*) Prove the De Moivre's theorem for the value of n, positive or negative, i.e. $((\cos\theta + \sqrt{-1}\sin\theta)^n = \cos n\theta + \sqrt{-1}\sin n\theta$.

(*b*) Expand $\sin^7\theta$ in series of series of multiples of θ.

(*c*) Separate into real and imaginary part of the quantity :

$\sin^{-1}(\cos\theta + i\sin\theta)$, where θ is real.