This question paper contains **5** printed pages]

R—68—2017

FACULTIES OF ARTS AND SCIENCE

B.A./B.Sc. (First Year) (First Semester) EXAMINATION

MARCH/APRIL, 2017

(CBCS/CGPA)

MATHEMATICS

Paper I

(Differential Calculus)

(MCQ+Theory)

(Saturday, 1-4-2017) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) *One* mark to each correctly answered MCQ.

(*iii*) Negative marking system is applicable.

(*iv*) Use black ball point pen to darken the circle of correct choice in OMR answer-sheet. Circle once darkened is final. No change is permitted.

(*v*) Darken only one circle for the answer of an MCQ.

(MCQs)

1. Choose the *correct* alternative for each of the following : 10

(*i*) The derivative of coth x, $x \in$ R is :

(*a*) $\text{cosech}^2\ x$ (*b*) $-\ \text{cosech}^2\ x$

(*c*) $\text{sech}^2\ x$ (*d*) $-\text{sech}^2\ x$

(*ii*) $\dfrac{d^n(a^{mx})}{dx^n} =$

(*a*) $m^n d^{mx}(\log\ a)^n$ (*b*) $m^n a^{mx}(\log\ a)$

(*c*) $m\ a^{mx}(\log\ a)^n$ (*d*) $m^n a^x(\log\ a)^n$

P.T.O.

(*iii*) The equation of the normal at a point '*t*' of the curve $x = f(t)$, $y = f(t)$ is :

(*a*) $[X - f(t)]f'(t) - [y - F(t)]\,F'(t) = 0$

(*b*) $[X - f(t)]F'(t) - [y - F(t)]\,f'(t) = 0$

(*c*) $[X - f(t)]F'(t) + [y - F(t)]\,f'(t) = 0$

(*d*) $[X - f(t)]f'(t) + [y - F(t)]\,F'(t) = 0$

(*iv*) The length of the sub-tangent at any point of the curve $y = f(x)$ is :

(*a*) $y\sqrt{\left[1 + \left(\frac{\partial y}{\partial x}\right)^2\right]}$ (*b*) $y\sqrt{\left[1 + \left(\frac{\partial x}{\partial y}\right)^2\right]}$

(*c*) $y\frac{dx}{dy}$ (*d*) $y\frac{dy}{dx}$

(*v*) If a function f is :

(*i*) Continuous in a closed interval $[a, b]$

(*ii*) Derivable in the open interval $]a, b[$

(*iii*) $f(a) = f(b)$, then there exists at least one value 'c' $\in]a, b[$ such that :

(*a*) $f'(c) < 0$ (*b*) $f'(c) > 0$

(*c*) $f'(c) \neq 0$ (*d*) $f'(c) = 0$

(*vi*) Cauchy form of remainder after n terms in Taylor's theorem is :

(*a*) $\frac{h^n(1-\theta)^{n-p}}{p(n-1)!} f^n(a + \theta h)$

(*b*) $\frac{h^{n-1}(1-\theta)^{n-1}}{(n-1)!} f^n(a + \theta h)$

(*c*) $\frac{h^n}{n\,!} f^n(a + \theta h)$

(*d*) $\frac{h^n}{(n-1)\,!} f^n(a + \theta h)$

(*vii*) Consider :

(*i*) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(*ii*) $\lim_{x \to 0} \frac{\tan x}{x} = 1$,

then :

(*a*) both (*i*), (*ii*) are true

(*b*) (*i*) is false (*ii*) is true

(*c*) (*i*) is true (*ii*) is false

(*d*) both (*i*), (*ii*) are false

(*viii*) $\lim_{k \to 0} \frac{f(a, b+k) - f(a, b)}{k}$, if it exists is called the partial derivative of $f(x, y)$ w.r.t. y at (a, b) and is denoted by :

(*a*) $f_{yx}(a, b)$ (*b*) $f_{xy}(a, b)$

(*c*) $f_y(a, b)$ (*d*) $f_x(a, b)$

(*ix*) If $z = \log(x^2 + y^2)$, then $\frac{\partial z}{\partial x} =$

(*a*) $\frac{x}{(x^2 + y^2)}$ (*b*) $\frac{x}{(x^2 + y^2)^2}$

(*c*) $\frac{2x}{(x^2 + y^2)^2}$ (*d*) $\frac{2x}{x^2 + y^2}$

(*x*) If $\sin^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right) = z$, then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} =$

(*a*) $2 \tan z$ (*b*) $\frac{1}{2} \tan z$

(*c*) $2 \sin z$ (*d*) $\frac{1}{2} \sin z$

P.T.O.

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) Prove that :

$$\frac{d^n[e^{ax}\sin(bx+c)]}{dx^n} = r^n e^{ax}\sin(bx+c+n\phi)$$

where $r = \sqrt{(a^2+b^2)}$, $\phi = \tan^{-1}\left(\frac{b}{a}\right)$.

(*b*) If u and v are two functions of x possessing derivative of the nth order, then prove that :

$$(uv)_n = u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + \ldots\ldots\ldots\ldots + {}^nC_r u_{n-r} v_r + \ldots\ldots + {}^nC_n u v_n.$$

(*c*) Find the equations of the tangent and normal at $\theta = \frac{\pi}{2}$ to the curve $x = a(\theta + \sin\theta)$, $y = a(1+\cos\theta)$.

3. Attempt any *two* of the following : 5 each

(*a*) If a function f is (*i*) continuous in a closed interval $[a, b]$ and (*ii*) derivable in the open interval $]a, b[$, then prove that there exists at least one value $c \in\,]a, b[$ such that :

$$\frac{f(b)-f(a)}{b-a} = f'(c).$$

(*b*) If in the Cauchy's mean value theorem, $f(x) = e^x$ and $F(x) = e^{-x}$, show that c is arithmetic mean between a and b.

(*c*) Evaluate :

$$\lim_{x\to 0}(\cos x)^{\cot x}.$$

4. Attempt any *two* of the following : 5 each

(*a*) If $z = f(x, y)$ is homogeneous function of x, y of degree n, then prove that :

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \, \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)\, z.$$

(*b*) If $u = 3(lx + my + nz)^2 - (x^2 + y^2 + z^2)$ and $l^2 + m^2 + n^2 = 1$, show that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

(*c*) If

$$u = \tan^{-1} \frac{x^3 + y^3}{x - y}, \; x \neq y,$$

show that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u.$$