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R—81—2017

FACULTIES OF ARTS/SCIENCE

B.A./B.Sc. (First Year) (First Semester) EXAMINATION

MARCH/APRIL, 2017

(CBCS/CGPA)

MATHEMATICS

Paper II

(Algebra and Trigonometry)

(MCQ+Theory)

(Wednesday, 5-4-2017) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) First 30 minutes for Question No. **1** (MCQ) and remaining time for theory.

(*iii*) Negative marking system for MCQs is applicable.

(*iv*) Use black ball point pen to darken the circle of correct answer in OMR answer-sheet. Circle once darkened is final.

(*v*) Figures to the right indicate full marks.

(MCQs)

1. Choose the most *correct* alternative : 1 each

(*i*) Which of the following statements is *not* true :

(*a*) Determinant of unit matrix is unit matrix

(*b*) Inverse of unit matrix is unit matrix

(*c*) Transpose of unit matrix is unit matrix

(*d*) Adjoint of unit matrix is unit matrix

(*ii*) If A and B are two square matrices of same order then which of the following is true :

(*a*) $|A + B| = |A| + |B|$

(*b*) $|AB| = |A|\ |B|$

(*c*) $|A - B| = |A| - |B|$

(*d*) All of the above

P.T.O.

(*iii*) The determinant of orthogonal matrix is ....

(*a*) 1 only (*b*) –1 only

(*c*) 1 or –1 (*d*) Neither 1 nor –1

(*iv*) If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, then the matrix A is ....

(*a*) Idempotent (*b*) Involutary

(*c*) Nilpotent (*d*) Orthogonal

(*v*) If I_n is an identity matrix of order n, then $\rho(I_n)$ = ...

(*a*) 1 (*b*) $n + 1$

(*c*) $n - 1$ (*d*) n

(*vi*) For what value of λ, the system $\begin{bmatrix} 1 & 2 \\ 3 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ have a unique solution :

(*a*) $\lambda = 6$ (*b*) $\lambda \neq 6$

(*c*) $\lambda \neq 0$ (*d*) for all λ

(*vii*) The roots of the equation $|A - \lambda I| = 0$, where A is a square matrix and I is identity matrix, are called ....

(*a*) eigen values (*b*) characteristic roots

(*c*) characteristic values (*d*) all of these

(*viii*) The Polar form of complex number $1 + \sqrt{-1}$ is :

(*a*) $\sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$ (*b*) $\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(*c*) $2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ (*d*) $\sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(*ix*) If $x + \frac{1}{x} = 2\cos\theta$, then x = ...

(*a*) $\cos\theta$ (*b*) $\sin\theta$

(*c*) $\cos\theta + i\sin\theta$ (*d*) $\sin\theta + i\cos\theta$

(*x*) Which of the following is true ?

(*a*) $\cosh^2 x - \sinh^2 x = 1$ (*b*) $\cosh^2 x + \sinh^2 x = 1$

(*c*) $\sinh^2 x - \cosh^2 x = 1$ (*d*) $\cosh^2 x + \sinh^2 x = -1$

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) If A be a *n*-square matrix, then prove that :

$$A(\text{adj. } A) = (\text{adj } A)\, A = |A|\, I_n .$$

(*b*) Prove that, inverse of a square matrix, if it exists is unique.

(*c*) By using principle of mathematical induction prove that if :

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}, \text{ then}$$

$$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}, \; n \text{ being positive integer.}$$

3. Attempt any *two* of the following : 5 each

(*a*) A system AX = B of *m* linear equations in *n* unknowns is consistent if and only if the coefficient matrix A and the augmented matrix [A : B] of the system have the same rank.

(*b*) Prove that A system AX = B of *n* non-homogeneous equations in *n* unknowns has a unique solution provided A is non-singular *i.e.* $\rho(A) = n$.

P.T.O.

(*c*) Find a row echelon matrix which is row equivalent to :

$$A = \begin{bmatrix} 0 & 0 & -2 & 3 & 1 \\ 2 & 4 & 1 & 4 & 3 \\ 1 & 2 & -3 & 1 & 2 \\ 4 & 8 & 2 & 3 & 5 \end{bmatrix}$$

and find $\rho R(A)$.

4. Attempt any *two* of the following : 5 each

(*a*) Prove the De Moivre's theorem for the value of n, positive or negative :

i.e. $\left(\cos\theta + \sqrt{-1}\sin\theta\right)^n = \cos n\theta + \sqrt{-1}\sin n\theta$.

(*b*) Expand $\cos^8\theta$ in a series of cosines of multiples of θ.

(*c*) Separate into its real and imaginary part of the quantity $\tan^{-1}(\alpha + \beta i)$.