This question paper contains **5** printed pages]

V—65—2017

FACULTY OF ARTS AND SCIENCE

B.A./B.Sc. (First Year) (First Semester) EXAMINATION

OCTOBER/NOVEMBER, 2017

(CBCS/CGPA Pattern)

MATHEMATICS

Paper I

(Differential Calculus)

(MCQ + Theory)

(Saturday, 11-11-2017) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) One mark to each correctly answered MCQ.

(*iii*) Negative marking system is applicable for MCQs.

(*iv*) Use black ball point pen to darken the circle of OMR answer sheet. Circle once darkened is final. No change is permitted.

(*v*) Darken only one circle for the answer of an MCQ.

MCQ

1. Choose the correct alternative for each of the following : 1 each

(1) The derivative of $\cosh x \ (x \in \mathrm{R})$ is :

(A) $\cosh x$ (B) $\sinh x$

(C) $-\cosh x$ (D) $-\sinh x$

(2) $\dfrac{d^n}{dx^n}\left(a^{mx}\right) =$

(A) $a^n . m^n . a^{mx}$ (B) $a^m . n^m . a^{mx}$

(C) $m^n . a^{mx} . (\log a)^n$ (D) $m^n . a^{mx} . (\log a)^m$

P.T.O.

(3) The equation of the tangent at the point 't' of the curve $x = f(t)$, $y = F(t)$ is :

(A) $[X - f(t)] F'(t) - [Y - F(t)] f'(t) = 0$

(B) $[X - f(t)] F'(t) + [Y - F(t)] f'(t) = 0$

(C) $[X - f(t)] f'(t) - [Y - F(t)] F'(t) = 0$

(D) $[X - f(t)] f'(t) + [Y - F(t)] F'(t) = 0$

(4) The length of the subnormal at any point of the curve $y = f(x)$ is :

(A) $x \frac{dy}{dx}$ (B) $y \frac{dy}{dx}$

(C) $x \frac{dx}{dy}$ (D) $y \frac{dx}{dy}$

(5) Rolle's theorem is applicable to :

(A) $f(x) = x^2$ in [–1, 1]

(B) $f(x) = x(x+3)\, e^{-x/2}$ in [–3, 0]

(C) Both (A) and (B)

(D) Neither (A) nor (B)

(6) Schlomilch and Roche form of remainder in Taylor's theorem is :

(A) $R_n = \frac{h^n (1-\theta)^{n-p}}{p(n-1)!} f^n (a + \theta h)$

(B) $R_n = \frac{h^n (1-\theta)^{n-1}}{(n-1)!} f^n (a + \theta h)$

(C) $R_n = \frac{h^n}{n!} f^n (a + \theta h)$

(D) $R_n = \frac{h^n}{n!} f^n (a)$

(7) $\lim_{x \to 0} x \log x =$

(A) -1 (B) 0

(C) 1 (D) ∞

(8) The partial derivative of $f(x, y)$ w.r.t. x at (a, b), denoted by $f_x(a\,b)$ is given by :

(A) $\lim_{h \to 0} \frac{f(a+h,\, b) - f(a,\, b+h)}{h}$, if it exists

(B) $\lim_{h \to 0} \frac{f(a+h,\, b) - f(a,\, b)}{h}$, if it exists

(C) $\lim_{h \to 0} \frac{f(a,\, b+h) - f(a,\, b)}{h}$, if it exists

(D) $\lim_{h \to 0} \frac{f(a+h,\, b+h) - f(a,\, b)}{h}$, if it exists

(9) If $f(x, y) = e^{x-y}$, then $\frac{\partial f}{\partial y} =$

(A) e^{x-y} (B) $(x-y)\, e^{x-y}$

(C) $-e^{x-y}$ (D) -1

(10) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then $\frac{\partial u}{\partial x} =$

(A) $\frac{3x^2 - 3yz}{x^3 + y^3 + z^3 - 3xyz}$

(B) $\frac{x^2 - yz}{x^3 + y^3 + z^3 - 3xyz}$

(C) $\frac{3x^2 + 3y^2 + 3z^2}{x^3 + y^3 + z^3 - 3xyz}$

(D) $\frac{x^2 + y^2 + z^2 - xyz}{x^3 + y^3 + z^3 - 3xyz}$

P.T.O.

Theory

2. Attempt any *two* of the following : 5 each

(*a*) Prove that :

$$\frac{d^n}{dx^n}[e^{ax}.\sin(bx+c)] = r^n.\, e^{ax}\sin(bx+c+n\phi)$$

where $r=\sqrt{a^2+b^2}$, $\phi=\tan^{-1}\left(\frac{b}{a}\right)$.

(*b*) If *u* and *v* are two functions of *x* possessing derivatives of the *n*th order, then prove that :

$$(uv)_n = u_n v + {}^n c_1\, u_{n-1} v + {}^n c_2\, u_{n-2}\, v_2 + \ldots\ldots\ldots\ldots$$

$$+ {}^n c_r u_{n-r} v_r + \ldots\ldots\ldots + {}^n c_n u v_n$$

(*c*) Show that in the case of the curve $\beta y^2 = (x+\alpha)^3$, the square of the subtangent varies as the subnormal.

3. Attempt any *two* of the following : 5 each

(*a*) If a function *f* is (*i*) continuous in a closed interval [*a*, *b*] and (*ii*) derivable in the open interval (*a*, *b*), then prove that there exists at least one value $c \in (a, b)$ such that :

$$\frac{f(b)-f(a)}{b-a} = f'(c).$$

(*b*) If in the Cauchy's mean value theorem, $f(x)=e^x$ and $F(x)=e^{-x}$, then show that *c* is arithmetic mean between *a* and *b*.

(*c*) Determine :

$$\lim_{x \to \pi/2} \frac{\tan 3x}{\tan x}.$$

4. Attempt any *two* of the following : 5 each

(*a*) If $Z = f(x, y)$ be a homogeneous function of *x, y* of degree *n* then prove that :

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = nz, \ \forall \ x, y \in$$

the domain of the function.

(*b*) If

$$u = x^2 . \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right), \ xy \neq 0$$

then prove that :

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}.$$

(*c*) If

$$u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right), \ x \neq y,$$

then prove that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u.$$