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AO—64—2018

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (First Year) (First Semester) EXAMINATION

MARCH/APRIL, 2018

(CBCS/CGPA Pattern)

MATHEMATICS

(Differential Calculus—I)

(MCQ & Theory)

(Monday, 26-03-2018) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes for Q. No. **1** and remaining for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball point pen to darken the circles on OMR sheet for Q. No. **1**.

(*v*) Negative marking system is applicable for Q. No. **1**.

MCQ

1. Choose the correct alternative for each of the following : 1 each

(*i*) If $y = e^{mx}$, then $y_n =$

(*a*) me^{mx} (*b*) $m^n e^{mx}$

(*c*) mne^{mx} (*d*) $m^n.e^{m^n.x}$

(*ii*) If $y = \log(\sin x)$, then $y_3 =$

(*a*) $\dfrac{2\cos x}{\sin^3 x}$ (*b*) $2\,\text{cosec}^2\, x.\cot x$

(*c*) $\dfrac{2\,\text{cosec}^2\, x}{\tan x}$ (*d*) All of these

(*iii*) The derivative of tanh x w.r.t. x is :

(*a*) $\text{sech}^2\, x$ (*b*) $-\text{sech}^2\, x$

(*c*) $\text{cosech}^2\, x$ (*d*) $-\text{cosech}^2\, x$

P.T.O.

(*iv*) The length of the normal to the given curve at any point (x, y) is :

(*a*) $\sqrt{1+\left(\frac{dx}{dy}\right)^2}$ (*b*) $x\sqrt{1+\left(\frac{dx}{dy}\right)^2}$

(*c*) $y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ (*d*) $\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

(*v*) If a function f is :

(*i*) continuous in the closed interval $[a, b]$

(*ii*) derivable in the open interval $]\ a, b\ [$ and

(*iii*) $f(a) = f(b)$, then there exist at least one value $c \in\ ]\ a, b\ [$ such that $f'(c) = 0$

This statement is known as :

(*a*) Lagrange's Mean Value Theorem

(*b*) Rolle's Theorem

(*c*) Liebnitz Theorem

(*d*) Cauchy's Mean Value Theorem

(*vi*) Which of the following is *not* an Indeterminate form ?

(*a*) $\infty + \infty$ (*b*) $\infty - \infty$

(*c*) $0.\infty$ (*d*) 0^{∞}

(*vii*) $\lim_{x \to 1} \frac{1+\log x - x}{1-2x+x^2}$:

(*a*) $\frac{1}{2}$ (*b*) $-\frac{1}{2}$

(*c*) 0 (*d*) 1

(*viii*) The partial derivative of f w.r.t. x, if exists, is given by :

(*a*) $\lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$ (*b*) $\lim_{k \to 0} \frac{f(x+h, y+k) - f(x+h, y)}{k}$

(*c*) $\lim_{hk \to 0} \frac{f(x+h, y+k) - f(x, y)}{hk}$ (*d*) $\lim_{k \to 0} \frac{f(x, y+k) - f(x, y)}{k}$

(ix) If $z = e^{ax} \sin by$, then $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ respectively are :

(a) $e^{ax} \sin by, e^{ax} \cos by$ (b) $ae^{ax}, be^{ax} \cos by$

(c) $ae^{ax} \sin by, be^{ax} \cos by$ (d) None of these

(x) If $z = e^{x-y}$, then $\frac{\partial^2 z}{\partial x^2}$ =

(a) e^{x-y} (b) $-e^{x-y}$

(c) $(x-y)e^{x-y}$ (d) $(1-y)e^{x-y}$

Theory

2. Attempt any *two* of the following : 5 each

(a) If $y = (ax+b)^m$, then prove that $y_n = \frac{mb}{(m-n)!} a^n.(ax+b)^{m-n}$

(b) If u and v are two functions of x, possessing derivatives of the nth order, then prove that :

$$(uv)_n = u_n v + {}^nC_1\, u_{n-1}\, v_1 + {}^nC_2\, u_{n-2}\, v_2 + \dots + {}^nC_r\, u_{n-r}\, v_r + \dots + {}^nC_n\, uv_n$$

(c) Find the equations of the tangent and normal at any point (x, y) of the curve $y = c \cosh (x/c)$.

3. Attempt any *two* of the following : 5 each

(a) If two functions $f(x)$ and $F(x)$ are derivable in $[a, b]$ and $F'(x) \neq 0$ for any value of in $[a, b]$, then prove that there exists at least one value 'c' of x belonging to $]\, a, b\, [$ such that :

$$\frac{f(b)-f(a)}{F(b)-F(a)} = \frac{f'(c)}{F'(c)}$$

(b) If $f(x) = (x-1)(x-2)(x-3)$; $x \in [0, 4]$, then find the value of c.

(c) Evaluate $\lim \left(\frac{\tan x}{x}\right)^{1/x}$.

P.T.O.

4. Attempt any *two* of the following : 5 each

(*a*) If $z = f(x, y)$ is a homogeneous function of x, y of degree n, then prove that :

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$$

$\forall x, y \in$ the domain of the function.

(*b*) If $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$; $x^2 + y^2 + z^2 \neq 0$, then show that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

(*c*) Verify Euler's theorem for $z = ax^2 + 2hxy + by^2$.