This question paper contains **4** printed pages]

B—84—2019

FACULTY OF ARTS AND SCIENCE

B.A./B.Sc. (First Year) (First Semester) EXAMINATION

MARCH/APRIL, 2019

(CBCS/CGPA Pattern)

MATHEMATICS

Paper I

(Differential Calculus)

(MCQ & Theory)

(Thursday, 28-3-2019) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) One mark to each correctly answered MCQ.

(*iii*) Negative marking system is applicable.

(*iv*) Use black pen to darken the circle of correct choice in OMR answer-sheet. Circle once darkened is final. No change is permitted.

(*v*) Darken only one circle for the answer of an MCQ.

MCQ

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) Which of the following is hyperbolic identity for $\sinh 2x =$ ....

(*a*) $\cosh 2x$ (*b*) $\text{cosech } 2x$

(*c*) $\sinh x \cdot \cosh x$ (*d*) $2\sinh x \cdot \cosh x$

(*ii*) If $y = a^{mx}$, then y_n is :

(*a*) $m^n a^{mx} \cdot (\log a)^n$ (*b*) $n^m a^{mx} \cdot (\log a)^m$

(*c*) $mx \cdot (a)^{mx - 1}$ (*d*) $a^{mn} \cdot \log a$

P.T.O.

(*iii*) The angle of intersection of two curves is defined as the angle between their :

(*a*) Normals (*b*) Radius vectors

(*c*) Tangents (*d*) None of these

(*iv*) The length of the subnormal to the curve $y = f(x)$ at any point is :

(*a*) $y\sqrt{1 + \left(\frac{dx}{dy}\right)^2}$ (*b*) $y \cdot \frac{dx}{dy}$

(*c*) $y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ (*d*) $y \cdot \frac{dy}{dx}$

(*v*) Which of the following not a condition of Rolle's theorem for a function $y = f(x)$ be :

(*a*) Continuous in $[a, b]$ (*b*) Derivable in (a, b)

(*c*) $f(a) = f(b)$ (*d*) $f(c) = 0$

(*vi*) The function $f(x) = x^3 - 3x^2 + 3x + 2$ is :

(*a*) Decreasing (*b*) Increasing

(*c*) Constant (*d*) All (*a*), (*b*), (*c*)

(*vii*) The limit of the function $\lim_{x \to 0} \frac{a^x - b^x}{x}$ is :

(*a*) 0 (*b*) $\log(a - b)$

(*c*) $\log\left(\frac{a}{b}\right)$ (*d*) Does not exist

(*viii*) A function f is said to be continuous if it is continuous at every point of its :

(*a*) Domain (*b*) Codomain

(*c*) Range (*d*) Set of real numbers

(*ix*) The functional equation $z = f(x, y)$ geometrically represents :

(*a*) a plane (*b*) a surface

(*c*) a line (*d*) a point

(*x*) If $x^x \cdot y^y \cdot z^z = c$, then $\frac{\partial z}{\partial x}$ is :

(*a*) $\frac{1 + \log x}{1 + \log z}$ (*b*) $-\frac{1 + \log x}{1 + \log z}$

(*c*) $-\frac{1}{x(1 + \log ex)}$ (*d*) $x \cdot \log(xyz)$

Theory

2. Attempt any *two* of the following : 5 each

(*a*) If u and v are two functions of x possessing derivative of the nth order, then prove that :

$$(uv)_n = u_n v + {}^n c_1 u_{n-1} v + {}^n c_2 u_{n-2} v_2 + \ldots\ldots + {}^n c_r u_{n-r} v_r + \ldots\ldots + {}^n c_n uv_n.$$

(*b*) Define the Hyperbolic functions, prove that :

$$\frac{d}{dx}\left(\tanh^{-1} x\right) = \frac{1}{1 - x^2}.$$

(*c*) Show that in the case of the curve $\beta y^2 = (x + \alpha)^3$, the square of subtangent varies as the subnormal.

3. Attempt any *two* of the following : 5 each

(*a*) If two functions $f(x)$ and $F(x)$ are derivable in $[a, b]$ and $F'(x) \neq 0$ for any value of x in $[a, b]$, then prove that there exists at least one value 'c' of x belonging to the open interval (a, b) such that :

$$\frac{f(b) - f(a)}{F(b) - F(a)} = \frac{f'(c)}{F'(c)}.$$

P.T.O.

(*b*) Show that :

$$\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2},$$

$0 < u < v$ and deduce that :

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}.$$

(*c*) Discuss the continuity of f at origin when :

$$f(x) = \frac{\log(\sin x)}{\cot x}$$

for $x = 0$ and $f(0) = 0$.

4. Attempt any *two* of the following : 5 each

(*a*) State and prove Euler's theorem on Homogeneous functions.

(*b*) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that :

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}.$$

(*c*) If

$$u = \cot^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right),$$

show that :

$$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} + \frac{1}{4}\sin 2u = 0.$$