This question paper contains **3** printed pages]

X—38—2019

FACULTY OF ARTS AND SCIENCE

B.A./B.Sc. (First Semester) (Regular) EXAMINATION

OCTOBER/NOVEMBER, 2019

(New Pattern)

MATHEMATICS

Paper II

(Algebra and Trigonometry)

(Friday, 18-10-2019) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

1. Define adjoint of a matrix. If A be an n-square matrix, then prove that : 15

$$A.(\text{adj}.A) = (\text{adj}.A) \,.\, A = |A| I_n.$$

Hence verify above for matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}.$$

Or

(*a*) Prove that, elementary operations do not alter the rank of a matrix. 8

(*b*) Reduce to raw reduced echelon form of the matrix : 7

$$A = \begin{bmatrix} 0 & 1 & 3 & -1 & 4 \\ 2 & 0 & -4 & 1 & 2 \\ 1 & 4 & 2 & 0 & -1 \\ 3 & 4 & -2 & 1 & 1 \\ 6 & 9 & -1 & 1 & 6 \end{bmatrix}$$

and find $\rho_R(A)$.

P.T.O.

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2. Define characteristic vectors. Prove that if λ is a characteristic root of a matrix A if and only if there exists a non-zero vector X such that $AX = \lambda X$.

Hence find the characteristic roots and the corresponding characteristic vectors

for the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -7 & 5 & 1 \end{bmatrix}$. 15

Or

(*a*) Expand $\cos^n \theta$ in a series of cosines of multiples of θ, n being a positive integer. 8

(*b*) If : 7

$$\sin \alpha + \sin \beta + \sin \gamma = \cos \alpha + \cos \beta + \cos \gamma = 0,$$

then prove that :

$$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos(\alpha + \beta + \gamma)$$

$$\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3 \sin(\alpha + \beta + \gamma)$$

3. Attempt any *two* of the following : 5 each

(*a*) Find the raw rank of matrix :

$$A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & 1 \\ 4 & 2 & 3 \end{bmatrix}.$$

(*b*) Prove that matrix multiplication is not commutative with suitable example.

(*c*) For what value of λ, the system :

$$\begin{bmatrix} 1 & 2 \\ 3 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ has

(*i*) a unique solution

(*ii*) more than one solution.

(*d*) Separate into its real and imaginary parts the expression $\tan(\alpha + i\beta)$.

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