This question paper contains 4 printed pages]

Y—102—2019

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (First Year) (First Semester) (Backlog) EXAMINATION

NOVEMBER/DECEMBER, 2019

(CBCS Pattern)

MATHEMATICS

Paper II

(Algebra and Trigonometry)

(MCQ + Theory)

(Thursday, 19-12-2019) **Time : 10.00 a.m. to 12.00 noon**

Time— Two Hours *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Negative marking system for MCQ is applicable.

(*iii*) Use black ball point pen to darken the circle of correct answer in OMR answer sheet. Circle once darken is final.

(*iv*) Figures to the right indicate full marks.

(MCQ)

1. Choose the most *correct* alternative for each of the following : 1 each

(*i*) If $A^2 = I$, then a square matrix A is said to be :

(*a*) Nilpotent Matrix (*b*) Involutory Matrix

(*c*) Idempotent Matrix (*d*) None of these

(*ii*) Which of the following is non-singular matrix ?

(*a*) $\begin{bmatrix} 4 & 2 \\ 6 & 3 \end{bmatrix}$ (*b*) $\begin{bmatrix} 1 & 7 \\ 9 & 2 \end{bmatrix}$

(*c*) $\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}$ (*d*) $\begin{bmatrix} -1 & 2 \\ -2 & 4 \end{bmatrix}$

P.T.O.

(*iii*) If A is an orthogonal matirx, then :

(*a*) $|A| = 1$ (*b*) $|A| = 0$

(*c*) $|A| = -1$ (*d*) Both (*a*) and (*c*)

(*iv*) If :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$$

then :

(*a*) $\rho(A) = 2$ (*b*) $\rho(A) = 0$

(*c*) $\rho(A) = 1$ (*d*) $\rho(A) = 3$

(*v*) The system :

$$x + y = 3$$
$$2x + 2y = 6$$

has :

(*a*) Unique solution (*b*) No soulution

(*c*) Infinitely many solutions (*d*) Only two solutions

(*vi*) A system AX = B of n non-homogeneous equations in n unknowns has a unique solution if :

(*a*) $\rho(A) = n$ (*b*) $|(A)| = 0$

(*c*) $\rho(A) \neq n$ (*d*) Both (*a*) and (*b*)

(*vii*) The number of non-zero rows in the column-echelon form of matrix A, is called :

(*a*) row rank of A (*b*) orthogonal matrix

(*c*) column rank of A (*d*) row echelon matrix

(*viii*) If Z = 1 + i, then modulus of Z is :

(*a*) 2 (*b*) $\sqrt{2}$

(*c*) –2 (*d*) $-\sqrt{2}$

(*ix*) If $x = \cos\theta + i\sin\theta$, then $x^n + \frac{1}{x^n}$ is equal to :

(*a*) $2\cos\theta$ (*b*) $2i\sin n\theta$

(*c*) $2i\cos n\theta$ (*d*) $2\cos n\theta$

(*x*) The series expansion of sin x is :

(*a*) $1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+............$

(*b*) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+............$

(*c*) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+............$

(*d*) $x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+............$

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) If A, B, C are matrices of the order $m \times n$, $n \times p$, $p \times q$ respectively, then prove that :

(AB) C = A (BC)

(*b*) Calculate the adjoint of A, where :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -1 & 2 \\ -2 & 1 & 1 \end{bmatrix}$$

(*c*) Find the inverse of the matrix A, given by :

$$A = \begin{bmatrix} 9 & 5 & 6 \\ 7 & -1 & 8 \\ 3 & 4 & 2 \end{bmatrix}$$

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) Prove that if λ is a characteristic root of a matrix A if and only if there exists a non-zero vector X such that AX = λX.

(*b*) Solve the system of equations :

$$x + 2y + 3z + 4t = 0$$
$$8x + 5y + z + 4t = 0$$
$$5x + 6y + 8z + t = 0$$
$$8x + 3y + 7z + 2t = 0$$

(*c*) Discuss the consistency of system of equations :

$$5x_1 + 3x_2 + 14x_3 = 4$$
$$x_2 + 2x_3 = 1$$
$$x_1 - x_2 + 2x_3 = 0$$
$$2x_1 + x_2 + 6x_3 + 2$$

4. Attempt any *two* of the follownig : 5 each

(*a*) Expand $\cos^n\theta$ in a series of cosines of multiples of θ, *n* being a positive integer.

(*b*) If $\sin \alpha + \sin \beta + \sin \gamma = \cos \alpha + \cos \beta + \cos \gamma = 0$, then prove that :

$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos (\alpha + \beta + \gamma)$ and

$\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3 \sin (\alpha + \beta + \gamma)$

(*c*) Separate into its real and imaginary parts the expression $\tan (\alpha + \beta i)$