This question paper contains **5** printed pages]

Y—84—2019

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (First Year) (First Semester) EXAMINATION

OCTOBER/NOVEMBER, 2019

(CBCS Pattern)

MATHEMATICS

Paper I

(Differential Calculus)

(MCQ + Theory)

(Monday, 18-11-2019) **Time : 10.00 a.m. to 12.00 noon**

Time— Two Hours *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) One mark to each correctly answered MCQ.

(*iii*) Negative marking system is applicable.

(*iv*) Use black ball pen to darken circle of correct choice on OMR answer sheet. Circle once darkened is final. No change is permitted.

(*v*) Darken only one circle for answer of an MCQ.

(MCQ)

1. Choose the *correct* alternative for each of the following : 10

(*i*) The derivative of cosech x, for all $x \in$ R is :

(*a*) coth x.cosech x (*b*) –coth x.cosech x

(*c*) –cosech^2 x (*d*) cosech^2 x

(*ii*) $\dfrac{d^n\left(\dfrac{1}{2x+3}\right)}{dx^n} = \ldots\ldots\ldots\ldots\ldots$

(*a*) $\dfrac{(1)^n n!(3)^n}{(2x+3)^{n+1}}$ (*b*) $\dfrac{(-1)^n n!(2)^n}{(2x+3)^n}$

(*c*) $\dfrac{(-1)^n (n-1)!(2)^n}{(2x+3)^n}$ (*d*) $\dfrac{(-1)^n n!(2)^n}{(2x+3)^{n+1}}$

P.T.O.

(*iii*) The equation of the tangent at the point 't' of the curve $x = f(t)$, $y = F(t)$ is ....

(*a*) $[X - f(t)]\ f'(t) + [Y - F'(t)]F(t) = 0$

(*b*) $[X - f(t)]\ F'(t) - [Y - F'(t)]f'(t) = 0$

(*c*) $[X - f(t)]\ F'(t) + [Y - F'(t)]f'(t) = 0$

(*d*) $[X + f(t)]\ f'(t) + [Y + F'(t)]F(t) = 0$

(*iv*) The length of the tangent at any point of the curve $y = f(x)$ is..........

(*a*) $y.\sqrt{\left[1+\left(\frac{dx}{dy}\right)^2\right]}$ (*b*) $y \cdot \frac{dx}{dy}$

(*c*) $y \cdot \frac{dy}{dx}$ (*d*) $y \cdot \sqrt{\left[1+\left(\frac{dy}{dx}\right)^2\right]}$

(*v*) If two functions $f(x)$ and $F(x)$ are derivable in closed interval $[a, b]$ and $F'(x) \neq 0$ for any value of x in $[a, b]$, then there exists at least one value 'c' of x belonging to the open interval $]a, b[$ such that $\frac{f'(c)}{F'(c)} =$ ......

(*a*) $\frac{F(b)-F(a)}{f(b)-f(a)}$ (*b*) $\frac{F(a)-F(b)}{f(b)-f(a)}$

(*c*) $\frac{f(b)-f(a)}{F(b)-F(a)}$ (*d*) $\frac{f(b)-f(a)}{b-a}$

(*vi*) Consider :

(*i*) $\lim_{x\to 0}\frac{\sin x}{x}=1$,

(*ii*) $\lim_{x\to 0}\frac{\tan x}{x}=1$, then :

(*a*) Both (*i*), (*ii*) are true (*b*) (*i*) is false (*ii*) is true

(*c*) (*i*) is true, (*ii*) is false (*d*) Both (*i*), (*ii*) are false (*vii*) If $\lim_{x\to a} f(x) = 1; \lim_{x\to a} F(x) = \infty$, then $\lim_{x\to a}[f(x)^{F(x)}]$ has the indeterminate form................ .

(*a*) 1^∞

(*b*) $\frac{0}{0}$

(*c*) ∞^0

(*d*) None of these

(*viii*) The partial derivative of $f(x, y)$ with respect to y is ....

(*a*) $\lim_{h\to 0} \frac{f(x+h, y) - f(x, y)}{h}$

(*b*) $\lim_{k\to 0} \frac{f(x+h, y+k) - f(x+h, y)}{k}$

(*c*) $\lim_{k\to 0} \frac{f(x, y+k) - f(x, y)}{k}$

(*d*) $\lim_{k\to 0} \frac{f(x, y) - f(x, y+k)}{k}$

(*ix*) If $z = f(x, y)$ be a homogeneous function of x, y of degree n, then $x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} =$ .... $\forall$ x, y $\in$, the domain of the function :

(*a*) nz

(*b*) $n(n - 1)z$

(*c*) n^2z

(*d*) $(n + 1)z$

(*x*) If $f(x, y) = \frac{\sqrt{y} + \sqrt{x}}{y + x}$, then degree of this homogeneous function is ....

(*a*) $\frac{1}{2}$

(*b*) $-\frac{1}{2}$

(*c*) 1

(*d*) –1

P.T.O.

(Theory)

2. Attempt any *two* of the following : (5 each)

(*a*) Prove that :

$$\frac{d^n}{dx^n}[e^{ax}.\sin(bx+c)] = r^n.e^{ax}\sin(bx+c+n\phi),$$

where $r = \sqrt{a^2+b^2}, \phi = \tan^{-1}(b/a)$

(*b*) Define hyperbolic functions and prove that :

$$\frac{d(\sinh^{-1}x)}{dx} = \frac{1}{\sqrt{(1+x^2)}}$$

(*c*) Find the angle of intersection of the parabolas $y^2 = 4\,ax$ and $x^2 = 4\,by$,. at their point of intersection other than the origin.

3. Attempt any *two* of the following : (5 each)

(*a*) If a function f is (*i*) continuous in a closed interval $[a, b]$, (*ii*) derivable in the open interval $]a, b[$ and (*iii*) $f(a) = f(b)$, then for at least one value 'c' $\in\ ]a, b[$, prove that $f'(c) = 0$.

(*b*) If $f(x) = e^x$ and $F(x) = e^{-x}$, by using Cauchy's mean value theorem, show that c is arithmetic mean between a and b.

(*c*) Determine :

$$\lim(x-a)^{x-a} \text{ as } x \to a$$

4. Attempt any *two* of the following : (5 each)

(*a*) If $z = f(x, y)$ is a homogeneous function of x, y of degree n, then prove that :

$$x^2.\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x.\partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$

(*b*) If $u = \log(\tan x + \tan y + \tan z)$, prove that :

$$(\sin 2x) \cdot \frac{\partial u}{\partial x} + (\sin 2y)\frac{\partial u}{\partial y} + (\sin 2z)\frac{\partial u}{\partial z} = 2$$

(*c*) If $u = \tan^{-1}\frac{x^3 + y^3}{x - y}, x \neq y$, then

show that :

$$x\frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = \sin 2u.$$