This question paper contains **4+1** printed pages]

BF—58—2016

FACULTY OF SCIENCE

B.Sc. (First Year) (Second Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

MATHEMATICS

Paper III

(Integral Calculus)

(MCQ + Theory)

(Tuesday, 18-10-2016) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—10+30=40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes for Q. No. **1** (MCQ) and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball pen to darken the circle on OMR sheet for Q. No. **1**.

(*v*) Negative marking system is applicable for Q. No. **1** (MCQ).

MCQ

1. Choose the *correct* alternative for each of the following : 1 each

(1) The integration means :

(A) a process which is the inverse of differentiation

(B) The process of finding the integral

(C) Both (A) and (B)

(D) None of the above

(2) $\int xe^x dx =$

(A) $e^x(x-1)$ (B) $e^{-x}(x-1)$

(C) $e^x(1-x)$ (D) $e^{-x}(x+1)$

(3) Rational functions of $(ax+b)^{1/n}$ and x can be easily evaluated by the substitution of :

(A) $t^n = ax - b$ (B) $t^n = ax + b$

(C) $t^{-n} = ax - b$ (D) $t^{-n} = ax + b$

(4) $\int \sin^n x \, dx =$

(A) $\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$

(B) $\frac{\sin^{n-1} x \cos x}{n} - \frac{n-1}{n} \int \sin^{n-2} x \, dx$

(C) $-\frac{\sin^{n-1} x \cos x}{n} - \frac{n-1}{n} \int \sin^{n-2} x \, dx$

(D) $-\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$

(5) Integration of $\sec^{2/3} x \operatorname{cosec}^{4/3} x$ is :

(A) $3 \tan^{-1/3} x$ (B) $-3 \tan^{-1/3} x$

(C) $-3 \sec^{-1/3} x$ (D) $3 \sec^{1/3} x$

(6) $\int_{a}^{b} f(x)\, dx =$

(A) $\int_{-a}^{-b} f(x)\, dx + \int_{-b}^{-a} f(x)\, dx$

(B) $\int_{-a}^{b} f(x)\, dx + \int_{b}^{-a} f(x)\, dx$

(C) $\int_{a}^{b} f(x)\, dx = \int_{b}^{a} f(x)\, dx$

(D) $\int_{a}^{c} f(x)\, dx + \int_{c}^{b} f(x)\, dx$

(7) $\int \cot^4 x\, dx =$

(A) $\frac{1}{3}\cot^3 x + \cot x + x$

(B) $-\frac{1}{3}\cot^3 x + \cot^2 x + x$

(C) $-\frac{1}{3}\cot^3 x + \cot x + x$

(D) $\frac{1}{3}\cot^3 x + \cos^2 x + x$

(8) The double integral of $f(x, y)$ over the region A, is denoted by :

(A) $\iint_{A} f(x, y)\, dx$ (B) $\iint_{A} f(x, y)\, dy$

(C) $\iint_{A} f(x, y)\, dA$ (D) None of these

(9) $\int_0^3 \int_x^{4x-x^2} dxdy =$

(A) $4\frac{1}{3}$ (B) $4\frac{1}{2}$

(C) $4\frac{1}{5}$ (D) 0

(10) The beta function B(m, n), for $m > 0$, $n > 0$ is defined by the relation :

(A) $B(m, n) = \int_0^1 x^{m-1} (1-x)^{n-1}\, dx$

(B) $B(m, n) = \int_0^1 x^{m+1} (1-x)^{n-1}\, dx$

(C) $B(m, n) = \int_0^1 x^{m+1} (1+x)^{n-1}\, dx$

(D) $B(m, n) = \int_0^1 x^{m+1} (1+x)^{n+1}\, dx$

Theory

2. Attempt any *two* of the following : 5 each

(*a*) Prove that :

The integral of the product of two functions = (first function) × (integral of second) – (integral of {diff. coeff. of first × integral of second})

(*b*) Prove that :

$$\int x^{m+n}(a+bx^n)^p\, dx = \frac{x^{m+1}\,(a+bx^n)^{p+1}}{a(m+1)} - \frac{b(np+m+n+1)}{a(m+1)} \int x^{m+n}(a+bx^n)^p\, dx$$

(*c*) Integrate :

$$\frac{(x^2+x+2)}{(x-2)\,(x-1)}.$$

3. Attempt any *two* of the following : 5 each

(*a*) Prove that :

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx .$$

(*b*) Define definite integral and prove that :

$$\int_{-a}^{a} f(x) \, dx = 0 \text{ or } 2 \int_{0}^{a} f(x) \, dx .$$

(*c*) Integrate :

$$x^2 \sin 2x \cdot$$

4. Attempt any *two* of the following : 5 each

(*a*) Prove that :

$$\int_0^{\pi/2} \cos^{2m-1} \theta \sin^{2n-1} \theta \, d\theta = \frac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)} .$$

(*b*) Evaluate the triple integral of the function $f(x, y, z) = x^2$ over the region V enclosed by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 0$.

(*c*) Find the area included between the curve :

$$xy^2 = 4a^2(2a - x)$$

and its asymptote.