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R—58—2017

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (First Year) (Second Semester) EXAMINATION

MARCH/APRIL, 2017

(CBCS/CGPA)

MATHEMATICS

Paper III (MT-103)

(Integral Calculus)

(MCQ + Theory)

(Friday, 31-3-2017) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—10+30=40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Negative marking system for MCQ is applicable.

(*iv*) Use black ball point pen to darken the circle of correct answer in OMR answer-sheet. Circle once darkened is final.

MCQ

1. Choose the *correct* alternative for each of the following : 1 each

(1) The function which is integrated is called the ....

(A) Integration (B) Integrand

(C) Differentiation (D) None of these

(2) Integral of the product of two functions = ...

(A) First function × integral of second – integral of {diff. coeff. of first × integral of second}

(B) Second function × integral of first – integral of {diff. coeff. of first × integral of second}

(C) First function × integral of second + integral of {diff. coeff. of first × integral of second}

(D) Second function × integral of first + integral of {diff. coeff. of first × integral of second}

P.T.O.

(3) The integral of the type

$$\int f(x, (ax+b)^{1/n})\, dx$$

can be evaluated by the substitution is :

(A) $t^n = (ax+b)^{1/n}$ (B) $t = (ax+b)^n$

(C) $t^n = ax+b$ (D) $t = ax+b$

(4) $\int \text{cosec}^n x\, dx = \ldots\ldots\ldots\ldots$

(A) $-\dfrac{\text{cosec}^{n-2}x.\cot x}{n-1} + \dfrac{n-2}{n-1}\int \text{cosec}^{n-2} x\, dx$

(B) $\dfrac{\text{cosec}^{n-2}x.\cot x}{n-1} + \dfrac{n-2}{n-1}\int \text{cosec}^{n-2} x\, dx$

(C) $-\dfrac{\text{cosec}^{n-2}x.\cot x}{n-1} - \dfrac{n-2}{n-1}\int \text{cosec}^{n-2} x\, dx$

(D) $-\dfrac{\text{cosec}^{n-1}x.\cot x}{n-1} + \dfrac{n-2}{n-1}\int \text{cosec}^{n-2} x\, dx$

(5) For

$$\int_a^b f(x)\, dx$$

which of the following is true ?

(A) This is the definite integral from a to b

(B) a and b are called its lower and upper limits

(C) The interval (a, b) is called range of integration

(D) All of the above

(6) $\int_a^b x^2 \, dx = \ldots\ldots\ldots\ldots$

(A) $\frac{1}{3} a^3 - \frac{1}{3} b^3$ (B) $\frac{1}{3} b^3 - \frac{1}{3} a^3$

(C) $\frac{1}{3} a^3 + \frac{1}{3} b^3$ (D) $\frac{1}{3} b^3 + \frac{1}{3} a^3$

(7) The area A lying between the curves

$$y = f_1(x), \quad y = f_2(x), \; x = a \text{ and } x = b$$

is ....

(A) $\iint_A d\,A = \int_a^b \int_{f_1(x)}^{f_2(x)} dy\, dx$

(B) $\iint_A d\,A = \int_{f_1(x)}^{f_2(x)} \int_a^b dx\, dy$

(C) $\iint_A d\,A = \int_a^b \int_{f_1(x)}^{f_2(x)} dx\, dy$

(D) None of the above

(8) $\int_0^\infty t^{-3/4} e^{-t} \, dt = \ldots\ldots\ldots\ldots\ldots$

(A) $\Gamma\left(\frac{3}{4}\right)$ (B) $\Gamma\left(\frac{1}{4}\right)$

(C) $\Gamma\left(\frac{5}{4}\right)$ (D) $\Gamma\left(\frac{2}{4}\right)$

P.T.O.

(9) $\Gamma(6) = \ldots\ldots\ldots\ldots$

(A) 5 ! (B) 120

(C) Both (A) and (B) (D) None of these

(10) If A is a region bounded by the curves :

$$x = f_1(y),\ x = f_2(y),\ y = c,\ y = d$$

then :

$$\iint_A f(x, y)\, dA = \ldots\ldots\ldots\ldots$$

(A) $\int_c^d \left\{\int_{f_1(y)}^{f_2(y)} f(x, y)\, dx\right\} dy$

(B) $\int_c^d \left\{\int_{f_1(y)}^{f_2(y)} f(x, y)\, dy\right\} dx$

(C) $\iint_A f(x, y)\, dA = \int_{f_1(y)}^{f_2(y)} \left\{\int_c^d f(x, y)\, dx\right\} dy$

(D) $\iint_A f(x, y)\, dA = \int_c^d f(x, y)\, dx\, dy$

Theory

2. Attempt any *two* of the following : 5 each

(*a*) Derive the reduction formula for $\int \cos^n x\, dx$.

(*b*) Prove that :

$$\int x^m (a + bx^n)^p\, dx + \frac{x^{m+1}(a + bx^n)^p}{np + m + 1} + \frac{anp}{np + m + 1} \int x^m (a + bx^n)^{p-1}\, dx$$

(*c*) Integrate $\dfrac{x}{(x-1)^2 (x+2)}$.

3. Attempt any *two* of the following : 5 each

(*a*) If f is a continuous function of x in the finite domain $[a, b]$ and $\frac{d\,F(x)}{dx} = f(x)$, then prove that :

$$\lim_{n\to 0} h[f(a) + f(a+h) + f(a+2h) + \ldots\ldots + f\{a+(n-1)h\} = F(b) - F(a)$$

where $h = \frac{(b-a)}{n}$.

(*b*) Derive the reduction formula for $\int x^n \sin mx\, dx$.

(*c*) Integrate $\int \sin^{5/6} x \cos^3 x\, dx$.

4. Attempt any *two* of the following : 5 each

(*a*) Define Gamma function and prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

(*b*) Prove that :

$$\int_0^{\pi/2} \cos^{2m-1}\theta \sin^{2n-1}\theta\, d\theta = \frac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)}.$$

(*c*) Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.