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R—74—2017

FACULTY OF SCIENCE

B.Sc. (Second Semester) EXAMINATION

MARCH/APRIL, 2017

(CBCS/CGPA)

MATHEMATICS

Paper IV

(Geometry)

(MCQ+Theory)

(Monday, 3-4-2017) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes for Question No. **1** and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball pen to darken the circle on OMR-sheet for Q. No. **1**.

(*v*) Negative marking system is applicable for Q. No. **1** (MCQ).

(MCQs)

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) If α, β, γ be the angles which a line makes with the positive direction of the axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma =$ ....

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

(*ii*) The equation to a plane in normal form is :

(*a*) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ (*b*) $\frac{x}{l} + \frac{y}{m} + \frac{z}{n} = 1$

(*c*) $ax + by + cz = p$ (*d*) $lx + my + nz = p$

P.T.O.

(*iii*) The equation of the plane passes through the intersection of the planes :

$$x + y + z = 6 \text{ and } 2x + 3y + 4z + 5 = 0$$

and the point (1, 1, 1), then the value of k is ....

(*a*) $\frac{3}{14}$ (*b*) $\frac{2}{14}$

(*c*) 14 (*d*) 1

(*iv*) The foot of the perpendicular from a given point on a given straight line is called ....

(*a*) Orthogonal sphere (*b*) Orthogonal projection

(*c*) Shortest distance (*d*) Direction cosine

(*v*) To transform the equations

$$ax + by + cz + d = 0, a_1x + b_1y + c_1z + d_1 = 0$$

of a line to the symmetrical form, requires ....

(*a*) The direction ratios of the line

(*b*) The co-ordinates of any one point on it

(*c*) Both (*a*) and (*b*)

(*d*) (*a*) or (*b*)

(*vi*) If $u_1 = 0, v_1 = 0$ and $u_2 = 0, v_2 = 0$ be two straight lines, then the general equations of a straight line intersecting them both are ....

(*a*) $u_1 + \lambda_1 v_1 = 0$ and $u_2 + \lambda_2 v_2 = 0$

(*b*) $u_1 + \lambda_1 v_1 = 1$ and $u_2 + \lambda_2 v_2 = 1$

(*c*) $u_1 - \lambda_1 v_1 = 0$ and $u_2 - \lambda_2 v_2 = 0$

(*d*) $u_1 - \lambda_1 v_1 = 1$ and $u_2 - \lambda_2 v_2 = 1$

(*vii*) Length of the shortest distance between the lines :

$$\frac{x-3}{3}=\frac{y-8}{-1},\ \frac{z-3}{1} \text{ and } \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$$

is :

(*a*) $\frac{3}{\sqrt{30}}$ (*b*) $3\sqrt{30}$

(*c*) $\sqrt{30}$ (*d*) None of these

(*viii*) The centre of sphere

$$x^2+y^2+z^2-6x+8y-10z+1=0$$

is ....

(*a*) (3, 4, 5) (*b*) (–3, –4, 5)

(*c*) (3, –4, –5) (*d*) (3, –4, 5)

(*ix*) The circle through three given points lies entirely on any sphere through the :

(*a*) same three points

(*b*) different three points

(*c*) same four points

(*d*) different four points

(*x*) The two equations :

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0,$$

$$lx+my+nz=p$$

taken together represent a ....

(*a*) Plane (*b*) Line

(*c*) Circle (*d*) Sphere

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) If *l*, *m*, *n* are the direction cosines of a plane, then prove that :

$$l^2+m^2+n^2=1.$$

(*b*) Prove that the two point $A(x_1, y_1, z_1)$, $B(x_2, y_2\ z_2)$ lie on the same or different sides of the plane $ax+by+cz+d=0$ according as the expressions $ax_1+by_1+cz_1+d$, $ax_2+by_2+cz_2+d$ are of the same or different signs.

P.T.O.

(*c*) The direction cosines l, m, n of two lines are connected by the relations $l + m + n = 0, 2lm + 2ln - mn = 0$. Find them.

3. Attempt any *two* of the following : 5 each

(*a*) Find the length of the perpendicular from a given point $P(x_1, y_1, z_1)$ to a given line :

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}.$$

(*b*) Find two points on the line :

$$\frac{x-2}{1} = \frac{y+3}{-2} = \frac{z+5}{2}$$

one either side of (2, –3, –5) and at a distance 3 from it.

(*c*) Show that the line $x + 10 = (8 - y)/2 = z$ lies in the plane $x + 2y + 3z = 6$.

4. Attempt any *two* of the following : 5 each

(*a*) Find the equation of the sphere described on the segment joining the points $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$ as a diameter.

(*b*) Find the equation of the cylinder whose generators intersect the conic :

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, \; z = 0$$

and are parallel to the line

$$\frac{x}{l} = \frac{y}{m} = \frac{z}{n}.$$

(*c*) Find the equation of the sphere through the four points (4, –1, 2), (0, –2, 3), (–1, –5, –1), (2, 0, 1).