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V—56—2017

FACULTY OF SCIENCE/ARTS

B.Sc. (First Year) (Second Semester) EXAMINATION

OCTOBER/NOVEMBER, 2017

(CBCS/CGPA Pattern)

MATHEMATICS

Paper III (MT-103)

(Integral Calculus)

(MCQ & Theory)

(Friday, 10-11-2017) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Negative marking system for MCQ is applicable.

(*iv*) Use black ball point pen to darken the circle for correct answer in OMR answer-sheet. Circle once darkened is final.

MCQ

1. Choose the *correct* alternative for each : 1 each

(*i*) If $\frac{d}{dx}F(x) = f(x)$, we say that $F(x)$ is a/an :

(*a*) Derivative of $f(x)$ (*b*) Integration of $f(x)$

(*c*) Both (*a*) and (*b*) (*d*) None of these

(*ii*) If $f(x)$ and $\phi(x)$ be two functions of x, then $\frac{d}{dx}\{f(x)\cdot\phi(x)\}$ is :

(*a*) $f(x)\cdot\phi'(x) + f'(x)\cdot\phi(x)$ (*b*) $f'(x)\cdot\phi'(x) + f'(x)\cdot\phi(x)$

(*c*) $f(x)\cdot\phi(x) + f'(x)\cdot\phi'(x)$ (*d*) $f'(x)\cdot\phi'(x) + f'(x)\cdot\phi'(x)$

P.T.O.

(*iii*) The integration of $\frac{1}{x\sqrt{(x^2-1)}}$ is :

(*a*) $\sin^{-1} x$ (*b*) $\tan^{-1} x$

(*c*) $\sec^{-1} x$ (*d*) $\text{cosec}^{-1} x$

(*iv*) The integration of $\cot^n x$ is :

(*a*) $\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x\, dx$ (*b*) $-\frac{\cot^{n-1} x}{n-1} + \int \cot^{n-2} x\, dx$

(*c*) $\frac{\cot^{n-1} x}{n-1} + \int \cot^{n-2} x\, dx$ (*d*) $-\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x\, dx$

(*v*) $\int_a^b f(x)dx =$ ...., where $C \in [a, b]$,

(*a*) $\int_c^a f(x)dx + \int_c^b f(x)dx$ (*b*) $\int_a^c f(x)dx + \int_c^b f(x)dx$

(*c*) $\int_a^c f(x)dx - \int_c^b f(x)dx$ (*d*) None of these

(*vi*) Integration of $\tan^n x\, dx$ is :

(*a*) $\int \tan^{n-2} x \sec^2 x\, dx - \int \tan^{n-2} x\, dx$

(*b*) $-\int \tan^{n-2} x \sec^2 x\, dx - \int \tan^{n-2} x\, dx$

(*c*) $\int \tan^{n-2} x \sec^2 x\, dx + \int \tan^{n-2} x\, dx$

(*d*) $\int \tan^{n-1} \sec x\, dx - \int \tan x\, dx$

(*vii*) If ν be a region of the three-dimensional space, and $f(x, y, z)$ be a function of the independent variable x, y, z defined at every point in ν. Then the triple integral of the function $f(x, y, z)$ over the region ν is denoted as :

(*a*) $\iiint_V f(x, y, z)\,dx$ (*b*) $\iiint_V f(x, y, z)\,dy$

(*c*) $\iiint_V f(x, y, z)\,dv$ (*d*) $\iiint_V f(x, y, z)\,dz$

(*viii*) The gamma function, $\Gamma(x)$, for $x > 0$, is defined as :

(*a*) $\Gamma(y) = \int_0^\infty t^{x-1} e^{-t}\,dt$ (*b*) $\Gamma(x) = \int t^{x-1} e^{-t}\,dt$

(*c*) $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\,dt$ (*d*) $\Gamma(y) = \int_0^\infty t^{y-1} e^{-t}\,dy$

(*ix*) Beta function $B(m, n)$, for $m > 0$, $n > 0$, is defined as :

(*a*) $B(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$

(*b*) $B(m, n) = \int_1^0 x^{m-1}(1-x)^{1-n}\,dx$

(*c*) $B(m, n) = \int_0^1 x^{1-m}(1-x)^{n-1}\,dx$

(*d*) $B(m, n) = \int_1^0 x^{1-m}(1+x)^{1-n}\,dx$

P.T.O.

(x) The Dirichlet's integral is :

(a) $\iiint x^{p-1} y^{m-1} z^{n-1} dx\, dy\, dz = \dfrac{\Gamma(p)\Gamma(m)\Gamma(n)}{\Gamma(p+m+n-1)}$

(b) $\iiint x^{p-1} y^{m-1} z^{n-1} dx\, dy\, dz = \dfrac{\Gamma(p)\Gamma(m)\Gamma(n)}{\Gamma(p+m+n+1)}$

(c) $\iiint x^{p+1} y^{m+1} z^{n+1} dx\, dy\, dz = \dfrac{\Gamma(p)\Gamma(m)\Gamma(n)}{\Gamma(p+m+n+1)}$

(d) $\iiint x^{p+1} y^{m+1} z^{n+1} dx\, dy\, dz = \dfrac{\Gamma(p)\Gamma(m)\Gamma(n)}{\Gamma(p-m-n-1)}$

Theory

2. Attempt any *two* of the following : 5 each

(a) Evaluate :

$$\int \sin^n x\, dx,$$

by successive reduction.

(b) Find the reduction formula for :

$$\int x^{m-n} \left(a + bx^n\right)^p dx.$$

(c) Integrate $\dfrac{\left(x^2 + x + 2\right)}{(x-2)(x-1)}$.

3. Attempt any *two* of the following : 5 each

(a) Find the reduction formula for :

$$\int \sin^m x \cos^n x\, dx.$$

(*b*) Define definite integral and prove that :

$$\int_{-a}^{a} f(x)\,dx = 0.$$

(*c*) Integrate $\tan^4 x$.

4. Attempt any *two* of the following : 5 each

(*a*) If A is a region bounded by the curves $y = f_1(x)$, $y = f_2(x)$, $x = a$ and $x = b$, then prove that :

$$\iint_A f(x, y)\,dA = \int_a^b \left\{ \int_{f_1(x)}^{f_2(x)} f(x, y)\,dy \right\} dx,$$

where the integration with respect to y is performed first treating x as a constant.

(*b*) Prove that :

$$\int_a^b f(x)\,dx \times \int_c^d g(y)\,dy = \int_a^b \int_c^d f(x)\,g(y)\,dx\,dy.$$

(*c*) Evaluate :

$$\int_0^3 \int_2^1 xy(1 + x + y)\,dx\,dy.$$