This question paper contains **5** printed pages]

AO—55—2018

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (First Year) (Second Semester) EXAMINATION

MARCH/APRIL, 2018

(CBCS/CGPA Pattern)

MATHEMATICS

Paper III

(Integral Calculus)

(MCQ+Theory)

(Saturday, 24-3-2018) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Negative marking system for MCQ is applicable.

(*iv*) Use black ball point pen to darken the circle of correct answer in OMR answer sheet. Circle once darkened is final. No change is permitted.

(MCQ)

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) $\int \lambda \cos x \, d\lambda =$

(*a*) $\lambda \sin x + c$ (*b*) $-\lambda \sin x + c$

(*c*) $\frac{\lambda^2}{2} \cos x + c$ (*d*) $\frac{\lambda^2}{2} \sin x + c$

(*ii*) Which of the following standard forms is true ?

(*a*) $\int \cos x \, dx = \sin x$ (*b*) $\int \cosh x \, dx = \sinh x$

(*c*) Both (*a*) and (*b*) (*d*) Neither (*a*) nor (*b*)

P.T.O.

(*iii*) $\int \frac{x}{(x-3)\sqrt{(x+1)}}\, dx =$

(*a*) $2\sqrt{x+1} + \frac{3}{2}\log\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}$

(*b*) $2\sqrt{x-3} + \frac{3}{2}\log\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}$

(*c*) $2\sqrt{x+3} + \frac{2}{3}\log\sqrt{x+1}-2$

(*d*) $2\sqrt{x-3} + \frac{3}{2}\log\frac{\sqrt{x+1-2}}{\sqrt{x+1+2}}$

(*iv*) $\int \tan^3 x\, dx =$

(*a*) $\tan^2 x + \log\cos x$

(*b*) $\frac{\tan^2 x}{2} + \log\cos x$

(*c*) $\frac{\tan^2 x}{2} + \cos x + C$

(*d*) $\frac{\tan^2 x}{2} - \log\cos x$

(*v*) Which of the following statements of the definite integral is *correct* ?

(*a*) $\int_a^b f(x)\, dx = -[F(a) - F(b)]$

(*b*) $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

(*c*) Both (*a*) and (*b*)

(*d*) None of the above

(*vi*) $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx =$ ....

(*a*) $-3 \tan^{-1/3} x$ (*b*) $3 \tan^{-1/3} x$

(*c*) $-3 \tan^{1/3} x$ (*d*) $3 \tan^{1/3} x + c$

(*vii*) If A is a region bounded by the curves $y = f_1(x)$, $y = f_2(x)$, $x = a$ and $x = b$, then $\iint\limits_{A} f(x, y)\, dA =$ ....

(*a*) $\int \left\{ \int\limits_{f_1(x)}^{f_2(x)} f(x, y)\, dy \right\} dx$

(*b*) $\left\{ \int\limits_{a}^{b} \int\limits_{f_2(x)}^{f_1(x)} f(x, y)\, dy \right\} dx$

(*c*) $\int\limits_{a}^{b} \left\{ \int\limits_{f_1(x)}^{f_2(x)} f(x, y)\, dx \right\} \int dy$

(*d*) $\int\limits_{a}^{b} \left\{ \int\limits_{f_1(x)}^{f_2(x)} f(x, y)\, dy \right\} dx$

(*viii*) $\int\limits_{0}^{\infty} t^x e^{-t}\, dt =$ ....

(*a*) $\sqrt{(x+1)}$ (*b*) $x\sqrt{x}$

(*c*) Both (A) and (B) (*d*) None of these

(*ix*) The value of $\left\lceil (1/2) \right.$ is ....

(*a*) π (*b*) $\sqrt{\pi}$

(*c*) $\sqrt{\pi^2}$ (*d*) $\frac{1}{2}\left\lceil \frac{1}{2} \right.$

P.T.O.

(*x*) $\int_a^b f(x)\, dx \times \int_c^d g(y)\, dy =$

(*a*) $\int_a^b \int_c^d f(x)\, g(y)\, dx\, dy$

(*b*) $\int_a^b \int_c^d f(x)\, g(y)\, dy\, dx$

(*c*) $\int_a^b \int_c^d g(y)\, f(x)\, dx\, dy$

(*d*) $\int_a^b \int_c^d f(y)\, g(x)\, dx\, dy$

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) If $f(x)$ and $\phi(x)$ be two functions of x, then prove that :

$$\int f_1(x)\, f_2(x)\, dx = f_1(x) \int f_2(x)\, dx - \int \{ f_1'(x) \int f_2(x)\, dx \} dx.$$

(*b*) Show that :

$$\int x^m (a + bx^n)^p\, dx =$$

$$\frac{x^{m+1}\,(a + bx^n)^{p+1}}{a(m+1)} - \frac{b(np + m + n + 1)}{a(m+1)} \int x^{m+n}\,(a + bx^n)^p\, dx.$$

(*c*) Evaluate :

$$\int \sin^6 x\, dx.$$

3. Attempt any *two* of the following : 5 each

(*a*) Prove the reduction formula for $\sec^n x\, dx$.

(*b*) Define definite integral and prove that :

$$\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx.$$

(*c*) Evaluate :

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx.$$

4. Attempt any *two* of the following : 5 each

(*a*) Prove that :

$$\int_0^{\pi/2} \cos^{2m-1}\theta \sin^{2n-1}\theta \, d\theta = \frac{\Gamma(m)\,\Gamma(n)}{2\,\Gamma(m+n)}.$$

(*b*) Prove that :

$$B(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}.$$

(*c*) Evaluate :

$$\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dy \, dx \, dz.$$