This question paper contains 5 printed pages]

W—60—2018

FACULTY OF SCIENCE/ARTS

B.Sc. (First Year) (Second Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(CBCS/CGPA Course)

MATHEMATICS

Paper (MT-103)

(Integral Calculus-III)

(MCQ+Theory)

(Tuesday, 16-10-2018) **Time : 10.00 a.m. to 12.00 noon**

Time—*Two Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *all* questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Negative marking system for MCQs is applicable.

(*iv*) Use black ball point pen to darken the circle for correct answer in OMR answer-sheet. Circle once darkened is final.

MCQ

1. Choose the *correct* alternative for each of the following : 10

(*i*) The integration means :

(*a*) A process which is the inverse of differentiation

(*b*) The process of finding the integral

(*c*) Both (A) and (B)

(*d*) None of the above

P.T.O.

(*ii*) $\int f_1(x)\, f_2(x)\, dx =$ ....

(*a*) $f_1(x)\int f_2(x)dx + \int \{f_1'(x)\int f_2(x)dx\}dx$

(*b*) $f_1(x)\int f_2(x)dx - \int \{f_1'(x)\int f_2(x)dx\}dx$

(*c*) $f_2(x)\int f_1(x)dx - \int \{f_1'(x)\int f_2(x)dx\}dx$

(*d*) $f_1(x)\int f_2(x)dx - \int \{f_1'(x)\, f_2(x)\}dx$

(*iii*) Rational function of $(ax + b)^{1/n}$ and x can be easily evaluated by the substitution of :

(*a*) $t^n = ax + b$ (*b*) $t^n = ax - b$

(*c*) $t^{-n} = ax + b$ (*d*) $t^{-n} = ax - b$

(*iv*) $\int \sin^7 x\, dx =$ ....

(*a*) $-\cos x + \cos^3 x - \frac{3}{5}\cos^5 x - \frac{1}{7}\cos^7 x$

(*b*) $\cos x + \cos^3 x - \frac{3}{5}\cos^5 x - \frac{1}{7}\cos^7 x$

(*c*) $-\cos x - \cos^3 x - \frac{3}{5}\cos^5 x - \frac{1}{7}\cos^7 x$

(*d*) $-\cos x + \cos^3 x - \frac{3}{5}\cos^5 x + \frac{1}{7}\cos^7 x$

(*v*) $\int \tan^4 x\, dx =$ ....

(*a*) $\frac{1}{3}\tan^3 x - \tan x + x$ (*b*) $\frac{1}{3}\tan^3 x - \tan x - x$

(*c*) $\frac{1}{3}\tan^3 x + \tan x - x$ (*d*) $\frac{-1}{3}\tan^3 x - \tan x + x$

(*vi*) $\int_a^b f(x)\,dx$ = ..., where $c \in [a, b]$

(*a*) $\int_c^a f(x)\,dx + \int_c^b f(x)\,dx$

(*b*) $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

(*c*) $\int_a^c f(x)\,dx - \int_c^b f(x)\,dx$

(*d*) $\int_a^c f(x)\,dx + \int_b^c f(x)\,dx$

(*vii*) $\int_1^2 x\,dx =$ ....

(*a*) $\frac{5}{2}$

(*b*) $\frac{1}{2}$

(*c*) $\frac{3}{2}$

(*d*) $\frac{-3}{2}$

(*viii*) If A is a region bounded by the curves $y = f_1(x)$, $y = f_2(x)$, $x = a$ and $x = b$, then :

$$\iint_A f(x, y)\,dA = ...$$

(*a*) $\int_a^b \left\{ \int_{f_1(x)}^{f_2(x)} f(x,y)\,dy \right\} dx$

(*b*) $\int_a^b \left\{ \int_{f_2(x)}^{f_1(x)} f(x,y)\,dy \right\} dx$

(*c*) $\int_b^a \left\{ \int_{f_1(x)}^{f_2(x)} f(x,y)\,dy \right\} dx$

(*d*) $\int_a^b \left\{ \int_{f_1(x)}^{f_2(x)} f(x,y)\,dy \right\} dy$

(*ix*) If V be a region of the three-dimensional space and $f(x, y, z)$ be a function of the independent variable x, y, z defined at every point in V. Then the triple integral of the function $f(x, y, z)$ over the region V is denoted as ....

(*a*) $\iiint_V f(x, y, z)\,dx$

(*b*) $\iiint_V f(x, y, z)\,dy$

(*c*) $\iiint_V f(x, y, z)\,dv$

(*d*) $\iiint_V f(x, y, z)\,dz$

P.T.O.

(x) $\Gamma(6)$ = ...

(a) 6 ! (b) 5 !

(c) 6 (d) 5

Theory

2. Attempt any *two* of the following : 5 each

(a) Derive the reduction formula for :

$$\int \cos^n x \, dx.$$

(b) Prove that :

$$\int x^m(a+bx^n)^p \, dx = \frac{x^{m+1}(a+bx^n)^p}{np+m+1} + \frac{an\,p}{np+m+1}\int x^m(a+bx^n)^{p-1} \, dx$$

(c) Integrate :

$$\frac{(x^2+x+2)}{(x-2)(x-1)}.$$

3. Attempt any *two* of the following : 5 each

(a) Prove the reduction formula for $\int \operatorname{cosec}^n x \, dx$.

(b) If f is a continuous function of x in the finite domain $[a, b]$ and $\frac{d}{dx}F(x) = f(x)$, then prove that :

$$\lim_{n\to\infty} h[f(a) + f(a+h) + f(a+2h) + \ldots + f\{a+(n-1)h\}] = f(b) - f(a)$$

where $h = \frac{(b-a)}{n}$.

(*c*) Evaluate :

$$\int_0^{\pi} \sin^4 x \, dx.$$

4. Attempt any *two* of the following : 5 each

(*a*) Prove that let $f(\theta)$ be continuous for every value of θ in the domain (α, β). Then the area bounded by the curve $r = f(\theta)$ and the radii vectors $\theta = \alpha$, $\theta = \beta$ $(\alpha < \beta)$, is equal to $\frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$.

(*b*) Prove that :

$$\int_0^{\pi/2} \cos^{2m-1}\theta \sin^{2n-1}\theta \, d\theta = \frac{\Gamma(m)\,\Gamma(n)}{2\Gamma(m+n)}.$$

(*c*) Find the area included between the curve :

$$xy^2 = 4a^2(2a - x)$$

and its asymptote.