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B—71—2019

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (First Year) (Second Semester) EXAMINATION

MARCH/APRIL, 2019

(CBCS/CGPA Pattern)

MATHEMATICS

Paper III (MT-103)

(Integral Calculus)

(MCQ+Theory)

(Wednesday, 27-3-2019) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Negative marking system for MCQs is applicable.

(*iv*) Use black ball point pen to darken the circle for correct answer in OMR answer-sheet.

(*v*) Choose most correct answer, circle once darkened is final.

(MCQ)

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) The function which is integrated is called :

(A) Integration (B) Integrand

(C) Differentiation (D) None of these

(*ii*) $\int e^x \sin x \, dx =$

(A) $\frac{1}{2}e^x (\sin x - \cos x)$ (B) $-\frac{1}{2}e^x (\sin x - \cos x)$

(C) $\frac{1}{2}e^x (\sin x + \cos x)$ (D) $-\frac{1}{2}e^x (\sin x + \cos x)$

P.T.O.

(*iii*) $\int \sin^n x \, dx =$

(A) $\left(\frac{1}{n}\right) \sin^{n-1} x \cos x + [(n-1)/n] \int \sin^{n-2} x \, dx$

(B) $\left(\frac{1}{n}\right) \sin^{n-1} x \cos x - [(n-1)/n] \int \sin^{n-2} x \, dx$

(C) $-\left(\frac{1}{n}\right) \sin^{n-1} x \cos x + [(n-1)/n] \int \sin^{n-2} x \, dx$

(D) $-\left(\frac{1}{n}\right) \sin^{n-1} x \cos x - [(n-1)/n] \int \sin^{n-2} x \, dx$

(*iv*) $\int \sec^{2/3} \operatorname{cosec}^{4/3} x \, dx =$

(A) $3 \tan^{-1/3} x$ (B) $3 \tan^{1/3} x$

(C) $-3 \tan^{-1/3} x$ (D) $-3 \operatorname{cosec}^{-1/3} x$

(*v*) $\int \tan^3 x \, dx =$

(A) $\frac{1}{2} \tan^2 x + \log \cos x$ (B) $\frac{1}{2} \tan^{-2} x - \log \cos x$

(C) $-\frac{1}{2} \tan^2 x + \log \cos x$ (D) $-\frac{1}{2} \tan^2 x - \log \cos x$

(*vi*) $\int_a^b f(x) \, dx =$

(A) $\int_b^a f(x) \, dx$ (B) $-\int_b^a f(x) \, dx$

(C) $[F(a) - F(b)]$ (D) None of these

(*vii*) If f is a continuous function of x in the finite domain $[a, b]$ and $dF(x)/dx = f(x)$, then $\lim_{n \to \infty} h[f(a) + f(a + h) + f(a + 2h) + \ldots\ldots\ldots\ldots +$ $f\{a + (n - 1)h\}] =$

(A) $f(b) - f(a)$, where $h = (b - a)/n$

(B) $F(b) + F(a)$, where $h = (b - a)/n$

(C) $F(b) - F(a)$, where $h = (b - a)/n$

(D) None of the above

(*viii*) If A is a region bounded by the curves $y = f_1(x)$, $y = f_2(x)$, $x = a$ and $x = b$, then $\iint_A f(x, y)\, dA = \ldots\ldots\ldots\ldots$ where the integration with respect to y is performed first treating x as a constant.

(A) $\int_b^b \left\{ \int_{f_1(x)}^{f_2(x)} f(x, y)\, dy \right\} dx$ (B) $\int_b^a \left\{ \int_{f_1(x)}^{f_2(x)} f(x, y)\, dy \right\} dx$

(C) $\int_a^b \left\{ \int_{f_2(x)}^{f_1(x)} f(x, y)\, dy \right\} dx$ (D) $\int_a^b \left\{ \int_{f_1(x)}^{f_2(x)} f(x, y)\, dy \right\} dx$

(*ix*) The gamma function $\Gamma(x)$, for $x > 0$, is defined by the relation :

(A) $\int_\infty^0 t^{x-1} e^{-t}\, dt$ (B) $\int_0^\infty t^{x-1} e^{-t}\, dt$

(C) $\int_0^\infty t^{x+1} e^{-t}\, dt$ (D) $\int_0^\infty t^{x+1} e^{t}\, dt$

(*x*) We define the beta function $B(m, n)$ for $m > 0$, $n > 0$, by the relation :

(A) $\int_0^1 x^{m-1}(1 - x)^{n-1}\, dx$ (B) $\int_0^1 x^{m+1}(1 - x)^{n-1}\, dx$

(C) $\int_0^1 x^{m-1}(1 + x)^{n-1}\, dx$ (D) $\int_0^1 x^{m+1}(1 + x)^{m+1}\, dx$

P.T.O.

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) Prove that integral of the product of two functions by integration by parts.

(*b*) Prove that :

$$\int x^m (a + bx^n)^p \, dx = \frac{x^{m+1} (a + bx^n)^{p+1}}{a(m+1)} - \frac{b(np + m + n + 1)}{a(m+1)} \int x^{m+n} (a + bx^n)^p \, dx.$$

(*c*) Integrate :

$$\frac{x}{(x-3)\sqrt{(x+1)}}.$$

3. Attempt any *two* of the following : 5 each

(*a*) Find the integration of $\sec^n x$.

(*b*) Prove :

$$\int_{-a}^{a} f(x)\, dx = 2\int_{0}^{a} f(x)\, dx$$

if $f(x)$ is an even function of x.

$= 0$, if $f(x)$ is an odd function of x.

(*c*) Integrate :

$$\frac{1}{\sin^3 x \cos^5 x}.$$

4. Attempt any *two* of the following : 5 each

(*a*) Prove that the area bounded by the curve $y = f(x)$, the axis of x, and the ordinates at $x = a$ and $x = b$ is given by :

$$\int_a^b y \, dx.$$

(*b*) Prove :

$$\Gamma(1/2) = \sqrt{\pi}.$$

(*c*) Evaluate :

$$\int_0^3 \int_1^2 xy\,(1 + x + y)\, dxdy.$$