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B—93—2019

FACULTY OF SCIENCE

B.Sc. (First Year) (Second Semester) EXAMINATION

MARCH/APRIL 2019

(CBCS/CGPA Pattern)

MATHEMATICS

Paper IV

(Geometry)

(MCQ & Theory)

(Friday, 29-3-2019) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First **30** minutes for Question No. **1** and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball pen to darken the circle on OMR sheet for Question No. **1**.

(*v*) Negative marking system is applicable for Question No. **1** (MCQ).

MCQ

1. Choose the *correct* alternative for each of the following : 10

(*i*) 6, 2, 3 are direction ratios of a line. What are the direction cosines ?

(*a*) $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}$ (*b*) $\frac{7}{6}, \frac{2}{7}, \frac{3}{7}$

(*c*) $\frac{6}{7}, \frac{7}{2}, \frac{3}{7}$ (*d*) $\frac{6}{7}, \frac{2}{7}, \frac{7}{3}$

P.T.O.

(*ii*) The foot of the perpendicular from a given point on a given straight line is called ....

(*a*) Direction ratios (*b*) Direction cosines

(*c*) Orthogonal projection (*d*) None of these

(*iii*) Two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d = 0$ are parallel if :

(*a*) $a_1a_2 + b_1b_2 + c_1c_2 = 0$ (*b*) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(*c*) $\frac{a_1}{a_2} + \frac{b_1}{b_2} + \frac{c_1}{c_2}$ (*d*) $a_1b_1 + a_2b_2 + c_1a_2 = 0$

(*iv*) Angle between the planes $x + y + z = 1$ and $x - y = 2$ is :

(*a*) $\frac{\pi}{4}$ (*b*) 0

(*c*) $\frac{\pi}{3}$ (*d*) $\frac{\pi}{2}$

(*v*) The number of arbitrary constants in the equations of a straight line is :

(*a*) 2 (*b*) 5

(*c*) 4 (*d*) 0

(*vi*) Any point on the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ is given by :

(*a*) $(r + 4, - 4r - 3, 7r - 1)$ (*b*) $(r - 4, - 4 + 3r, 7r + 1)$

(*c*) $(r + 4, - 4r + 3, -7r + 1)$ (*d*) $(r, - 4r - 3, 7r - 1)$

(*vii*) The straight line through (a, b, c) and parallel to y-axis is :

(*a*) $\frac{x-a}{0} = \frac{y-b}{1} = \frac{z-c}{0}$ (*b*) $\frac{x-a}{1} = \frac{y-b}{0} = \frac{z-c}{0}$

(*c*) $\frac{x-a}{0} = \frac{y-b}{0} = \frac{z-c}{1}$ (*d*) $\frac{x-a}{1} = \frac{y-b}{1} = \frac{z-c}{1}$

(*viii*) The centre of the sphere $2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z + 3 = 0$ is :

(*a*) (–2, 4, 2) (*b*) (–1, 2, 1)

(*c*) $\left(\frac{1}{2}, -1, -\frac{1}{2}\right)$ (*d*) $\left(\frac{-1}{2}, 1, \frac{1}{2}\right)$

(*ix*) The equation of a cone whose generators intersect a given conic or touch a given sphere is of :

(*a*) First degree (*b*) Second degree

(*c*) Fourth degree (*d*) None of these

(*x*) Guiding curve of a right circular cylinder is :

(*a*) Any closed curve (*b*) Ellipse

(*c*) Pair of straight lines (*d*) Circle

Theory

2. Attempt any *two* of the following : 5 each

(*a*) Prove that the projection of a segment AB on a line CD is AB.cos θ, where θ is the angle between the lines AB and CD.

(*b*) Prove that the two points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ lie on the same or different sides of the plane $ax + by + cz + d = 0$, according as the expressions

$$ax_1 + by_1 + cz_1 + d, \; ax_2 + by_2 + cz_2 + d$$

are of the same or different signs.

(*c*) Find the equation of the plane through the point (2, 3, 4) and parallel to the plane $5x - 6y + 7z = 3$.

3. Attempt any *two* of the following : 5 each

(*a*) Transform the equations

$$ax + by + cz + d = 0, \; a_1x + b_1y + c_1z + d_1 = 0$$

of a line to the symmetrical form.

P.T.O.

(*b*) Show that the line $x + 10 = \frac{(8 - y)}{2} = z$ lies in the plane $x + 2y + 3z = 6$ and the line $\frac{1}{3}(x - 2) = -(y + 2) = \frac{1}{4}(z - 3)$ in the plane

$$2x + 2y - z + 3 = 0.$$

(*c*) Show that the lines

$$\frac{x+5}{3} = \frac{y+4}{1} = \frac{z-7}{-2}, \quad 3x + 2y + z - 2 = 0 = x - 3y - 2z - 13$$

are coplanar and find the equation to the plane in which they lie.

4. Attempt any *two* of the following : 5 each

(*a*) Find the equation of the sphere described on the segment joining the points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ as a diameter.

(*b*) Find the equation of the cone whose vertex is the point (α, β, γ) and whose generators intersect the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0, \ z = 0.$$

(*c*) Obtain the equation of the sphere which passes through the three points (1, 0, 0), (0, 1, 0), (0, 0, 1) and has its radius as small as possible.