This question paper contains **4** printed pages]

Y—71—2019

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (First Year) (Second Semester) (Backlog) EXAMINATION

NOVEMBER/DECEMBER, 2019

(CBCS/CGPA Pattern)

MATHEMATICS

Paper III

(Integral Calculus)

(MCQ & Theory)

(Monday, 23-12-2019) **Time : 10.00 a.m. to 12.00 noon**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Negative marking system for MCQ is applicable.

(*iv*) Use black ball point pen to darken the circle of correct answer in OMR answer-sheet. Circle once darkened is final. No change is permitted.

MCQ

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) If $\frac{d}{dx}F(x) = f(x)$ we say that F(*x*) is a/an :

(*a*) Derivative of $f(x)$ (*b*) Integration of $f(x)$

(*c*) Both (*a*) and (*b*) (*d*) None of these

(*ii*) $\int xe^x dx =$

(*a*) $e^x(x - 1)$ (*b*) $e^{-x}(x - 1)$

(*c*) $e^x(1 - x)$ (*d*) $e^{-x}(x + 1)$

P.T.O.

(*iii*) Reduction formula for $\int \sin^n x\,dx =$

(*a*) $\dfrac{\sin^{n-1} x \cdot \cos x}{n} + \dfrac{n-1}{n} \int \sin^{n-2} x\,dx$

(*b*) $\dfrac{\sin^{n-1} x \cdot \cos x}{n} - \dfrac{n-1}{n} \int \sin^{n-2} x\,dx$

(*c*) $\dfrac{-\sin^{n-1} x \cdot \cos x}{n} - \dfrac{n-1}{n} \int \sin^{n-2} x\,dx$

(*d*) $\dfrac{-\sin^{n-1} x \cdot \cos x}{n} + \dfrac{n-1}{n} \int \sin^{n-2} x\,dx$

(*iv*) Integration of $\sec^{2/3} x \cdot \operatorname{cosec}^{4/3} x$ is :

(*a*) $3 \tan^{-1/3} x$ (*b*) $-3 \tan^{-1/3} x$

(*c*) $-3 \sec^{-1/3} x$ (*d*) $3 \sec^{1/3} x$

(*v*) If $\int f(x)dx = F(x)$, then $\int_a^b f(x)dx =$

(*a*) $F(b) - F(a)$ (*b*) $\int_a^b f(t)dt$

(*c*) $-\int_b^a f(x)dx$ (*d*) All of these

(*vi*) $\int_0^{\pi} \sin^4 x\,dx =$

(*a*) $-\dfrac{3\pi}{8}$ (*b*) $\dfrac{8\pi}{3}$

(*c*) $\dfrac{3\pi^2}{8}$ (*d*) $\dfrac{3\pi}{8}$

(*vii*) $\int x^n \cdot \sin mx \, dx =$

(*a*) $\dfrac{x^n \cos mx}{m} + \dfrac{nx^{n-1} \sin mx}{m^2} - \dfrac{n(n-1)}{m^2} \int x^2 \cdot \sin mx \, dx$

(*b*) $\dfrac{x^n \cos mx}{m} - \dfrac{nx^{n-1} \sin mx}{m^2} - \dfrac{n(n-1)}{m^2} \int x^{n-2} \sin mx \, dx$

(*c*) $-\dfrac{x^n \cdot \cos mx}{m} + \dfrac{nx^{n-1} \cdot \sin mx}{m^2} - \dfrac{n(n-1)}{m^2} \int x^{n-2} \sin mx \, dx$

(*d*) $-\dfrac{x^n \cdot \cos mx}{m} + \dfrac{nx^{n-1} \cdot \sin mx}{m^2} + \dfrac{n(n-1)}{m^2} \int x^{n-2} \sin mx \, dx$

(*viii*) The double integral of $f(x, y)$ over the region A, is denoted by :

(*a*) $\iint\limits_A f(x, y) dx$ (*b*) $\iint\limits_A f(x, y) dy$

(*c*) $\iint\limits_A f(x, y) dA$ (*d*) None of these

(*ix*) The value of $\overline{|(1/2)}$ is :

(*a*) π (*b*) $\sqrt{\pi}$

(*c*) $\sqrt{\pi^2}$ (*d*) $\dfrac{1}{2}\overline{|\dfrac{1}{2}}$

(*x*) The area of the circle $x^2 + y^2 = 1$ is :

(*a*) π (*b*) $\dfrac{\pi}{2}$

(*c*) $+\dfrac{\pi}{3}$ (*d*) $\dfrac{\pi}{4}$

Theory

2. Attempt any *two* of the following : 5 each

(*a*) Prove that :

$$\int \cos^n x \, dx = \frac{\cos^{n-1} x \cdot \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx.$$

P.T.O.

(*b*) If $f(x)$ and $\phi(x)$ be two functions of x, then prove that :

$$\int f_1(x) \cdot f_2(x)\, dx = f_1(x) \int f_2(x)\, dx - \int \left\{ f_1'(x) \int f_2(x)\, dx \right\} dx.$$

(*c*) Evaluate :

$$\int x^2 \cdot \sin x\, dx.$$

3. Attempt any *two* of the following : 5 each

(*a*) Prove that :

$$\int \sec^n x\, dx = \frac{\sec^{n-2} x \cdot \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x\, dx.$$

(*b*) Show that :

$$\int_{-a}^{a} f(x)\, dx = 0, \; f(x) \text{ is odd function of } x$$

$$= 2 \int_0^a f(x)\, dx, \; f(x) \text{ is an even function of } x.$$

(*c*) Evaluate :

$$\int \sin^{5/6} x \cdot \cos^3 x\, dx.$$

4. Attempt any *two* of the following : 5 each

(*a*) Prove that :

$$\int_a^b f(x)\, dx \times \int_c^d g(y)\, dy = \int_a^b \int_c^d f(x)\, g(y)\, dx \cdot dy.$$

(*b*) Prove that :

$$\int_0^{\pi/2} \cos^{2m-1} \theta \cdot \sin^{2n-1} \theta\, d\theta = \frac{\overline{|(m)}\, \overline{|(n)}}{2\overline{|(m+n)}}.$$

(*c*) Evaluate :

$$\int_0^3 \int_1^2 xy(1 + x + y)\, dx \cdot dy.$$