This question paper contains **4+1** printed pages]

BF—71—2016

FACULTY OF SCIENCE/ARTS

B.Sc./B.A. (Second Year) (Third Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

MATHEMATICS

Paper VI

(Real Analysis—I)

(MCQ + Theory)

(Wednesday, 19-10-2016) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :—(*i*) *All* questions are compulsory.

(*ii*) First 30 minutes for Q.No. **1** and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball pen to darken the circle on OMR sheet for Q. No. **1**.

(*v*) Negative marking system is applicable for Q. No. **1** (MCQ).

MCQ

1. Choose the *correct* alternative for each of the following : 1 each

(1) $f(x) = \sin x$ $(-\infty < x < \infty)$, the image of $\frac{\pi}{2}$ under f is ....

(*a*) 0 (*b*) 1

(*c*) –1 (*d*) $\frac{1}{2}$

(2) If a, b are real numbers, then $\max(a, b)$ = ...

(*a*) $\frac{|a-b|+a-b}{2}$ (*b*) $\frac{-|a-b|+a+b}{2}$

(*c*) $\frac{|a-b|+a+b}{2}$ (*d*) $\frac{|a-b|-a+b}{2}$

P.T.O.

(3) The binary expansion of $\frac{1}{2}$ is ....

(*a*) 0.10000 .... (*b*) 0.01000 ....

(*c*) 0.0001 ..... (*d*) 0.11010 .....

(4) If A is any non-empty subset of R that is bounded then A has .... in R.

(*a*) No least upper bound

(*b*) No greatest lower bound

(*c*) Greatest lower bound

(*d*) Least upper bounded

(5) If

$$C = \{C_n\}_{n=1}^{\infty} = \{\sqrt{n}\}_{n=1}^{\infty} \text{ and}$$

$$N = \{n_i\}_{i=1}^{\infty} = \{i^4\}_{i=1}^{\infty}$$

then C o N = ...

(*a*) $\{i^4\}_{i=1}^{\infty}$ (*b*) $\{i^2\}_{i=1}^{\infty}$

(*c*) $\{i\}_{i=1}^{\infty}$ (*d*) $\{i^6\}_{i=1}^{\infty}$

(6) Let $\{S_n\}_{n=1}^{\infty}$ be a sequence of real numbers. If

$$S_1 \le S_2 \le .... \le S_n \le S_{n+1} \le .....$$

then $\{S_n\}_{n=1}^{\infty}$ is called ....

(*a*) non-decreasing (*b*) decreasing

(*c*) non-increasing (*d*) increasing

(7) If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers which converges to L, then $\{S_n^2\}_{n=1}^{\infty}$ converges to ....

(*a*) L (*b*) 1

(*c*) L^2 (*d*) ∞

(8) If $0 < x < 1$, then $\sum_{n=0}^{\infty} x^n$ converges to ....

(*a*) 1 (*b*) $\frac{1}{1+x}$

(*c*) $\frac{1}{x}$ (*d*) $\frac{1}{1-x}$

(9) Let $\sum_{n=1}^{\infty} a_n$ be a series of real numbers. If $\sum_{n=1}^{\infty} |a_n|$ converges, we say that $\sum_{n=1}^{\infty} a_n$ ....

(*a*) Converges absolutely

(*b*) Diverges absolutely

(*c*) Converges conditionally

(*d*) Diverges conditionally

(10) If $\lim \sup_{n \to \infty} \sqrt[n]{|a_n|} = A$, then the series of real numbers $\sum_{n=1}^{\infty} a_n$ converges absolutely if ....

(*a*) $A = 1$ (*b*) $A < 1$

(*c*) $A > 1$ (*d*) $A = 0$

Theory

2. Attempt any *two* of the following : 5 each

(*a*) If $f : A \to B$ and if $X \subset B$, $Y \subset B$, then prove that :

$$f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y).$$

(*b*) If B is an infinite subset of the countable set A, then prove that B is countable.

(*c*) If A, B are subsets of S, then χ_A (called the characteristic function of A) then prove that :

$$\chi_{A \cap B} = \min(\chi_A, \chi_B) = \chi_A \chi_B.$$

3. Attempt any *two* of the following : 5 each

(*a*) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent, then prove that $\{S_n\}_{n=1}^{\infty}$ is bounded.

(*b*) If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n \to \infty} S_n = L$, and if $\lim_{n \to \infty} t_n = M$, then prove that :

$$\lim_{n \to \infty} (S_n + t_n) = L + M.$$

(*c*) Prove that :

(*i*) $$\lim_{n \to \infty} \frac{3n^2 - 6n}{5n^2 + 4} = \frac{3}{5}$$

(*ii*) $$\lim_{n \to \infty} \frac{2n^3 + 5n}{4n^3 + n^2} = \frac{1}{2}.$$

4. Attempt any *two* of the following : 5 each

(*a*) If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B, then prove that :

$$\sum_{n=1}^{\infty} (a_n + b_n)$$

converges to A + B. Also, if C ∈ R, then $\sum_{n=1}^{\infty} \mathrm{C}a_n$ converges to CA.

(*b*) If $\sum_{n=1}^{\infty} b_n$ converges absolutely and if $\lim_{n \to \infty} |a_n| / |b_n|$ exists, then prove that $\sum_{n=1}^{\infty} a_n$ converges absolutely.

(*c*) Prove that :

$$1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\ldots$$

converges.