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BF—86—2016

FACULTY OF SCIENCE

B.Sc. (Second Year) (Third Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

MATHEMATICS

Paper VII

(Group Theory)

(MCQ+Theory)

(Friday, 21-10-2016) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) First 30 minutes for Question No. **1** and remaining time for other questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use black pen to darken the circle on OMR sheet for Q. No. **1** (MCQ).

(*iv*) Negative marking system is applicable for Q. No. **1** (MCQ).

(MCQ)

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) Let $n > 0$ be a fixed integer and $a \equiv b \bmod n$, if :

(*a*) $n \mid (a + b)$ (*b*) $n \mid (a - b)$

(*c*) $n \mid (b - a)$ (*d*) $n \mid (b \div a)$

(*ii*) G is a group then which of the following is *true* ?

(*a*) $a, b \in G \Rightarrow a \,.\, b \in G$

(*b*) $a, b, c \in G \Rightarrow a \,.\, (b \,.\, c) = (a \,.\, b) \,.\, c$

(*c*) There exists an element $e \in G$ such that $a \,.\, e = e \,.\, a = a$ for all G

(*d*) All of the above

P.T.O.

(*iii*) Let G be the set of all 2 × 2 non-singular matrices then G is :

(*a*) infinite, non-abelian group

(*b*) finite, abelian group

(*c*) infinite, abelian group

(*d*) finite, non-abelian group

(*iv*) If H is a subgroup of G and K is a subgroup of H, then :

(*a*) K is empty set (*b*) K = G

(*c*) K is subgroup of G (*d*) G is subgroup of K

(*v*) If H is subgroup of G, $a \in G$, then which of the following is a right coset of H in G ?

(*a*) $Ha = \{ah \mid h \in H\}$ (*b*) $Ha = \{a^{-1}ha \mid h \in H\}$

(*c*) $Ha = \{aha^{-1} \mid h \in H\}$ (*d*) $Ha = \{ha \mid h \in H\}$

(*vi*) If H and K are finite subgroups of G of orders $o(H)$ and $o(K)$, respectively, then $O(HK)$ = ?

(*a*) $\dfrac{o(H)\, o(K)}{o(H \cap K)}$ (*b*) $o(H) \,.\, o(K)$

(*c*) $o(H \cap K)$ (*d*) $o(H) \,.\, o(K) \,.\, o(H \cap K)$

(*vii*) If ϕ is a homomorphism of G into $\overline{G}$, then $\phi(x^{-1}) = ?$

(*a*) $\phi(x)$ (*b*) $\phi(x)^{-1}$

(*c*) e (*d*) $\overline{e}$

(*viii*) If ϕ is a homomorphism of G into $\overline{G}$ with kernel K, then K is :

(*a*) a normal subgroup of G

(*b*) a normal subgroup of $\overline{G}$

(*c*) non-empty subset of $\overline{G}$

(*d*) None of the above

(*ix*) What is an automorphism of a group G ?

(*a*) It is onto

(*b*) It is isomorphism

(*c*) It is mapping from G to itself

(*d*) All of the above

(*x*) If G is a finite group and $a \in G$, then $a^{O(G)} = ?$

(*a*) a (*b*) e

(*c*) a^{-1} (*d*) O(G)

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) If a and b are integers, not both 0, then prove that (a, b) exists, moreover, we can find integers m_0 and n_0 such that $(a, b) = m_0 a + n_0 b$.

(*b*) If G is a group, then prove that :

(*i*) For every $a \in G, (a^{-1})^{-1} = a$

(*ii*) For all $a, b \in G, (a.b)^{-1} = b^{-1}. a^{-1}$.

(*c*) Prove that if G is an abelian group, then prove that for all $a, b \in G$ and all integers n, $(a . b)^n = a^n . b^n$.

3. Attempt any *two* of the following : 5 each

(*a*) Prove that N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \in G$.

(*b*) If G is a finite group and H is a subgroup of G, then prove that O(H) is a divisor of O(G).

(*c*) If H is a subgroup of G and N is a normal subgroup of G, show that $H \cap N$ is a normal subgroup of H.

4. Attempt any *two* of the following : 5 each

(*a*) Suppose G is a group N a normal subgroup of G, define the mapping ϕ from G to G/N by $\phi(x) = Nx$ for all $x \in G$, then prove that ϕ is a homomorphism of G onto G/N.

(*b*) Prove that every permutation is the product of its cycles.

(*c*) For two permutations :

$$\psi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$$

and $$\phi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix}$$

Compute $\psi\phi$ and $\phi\psi$.