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BF—366—2016

FACULTY OF SCIENCE

B.Sc. (Second Year) (Third Semester) EXAMINATION

NOVEMBER/DECEMBER, 2016

MATHEMATICS

Paper VIII (MT-203)

(Ordinary Differential Equations)

(MCQ + Theory)

(Thursday, 1-12-2016) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—10+30=40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes for Q. No. **1** and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball pen to darken the circle on OMR sheet for Q. No. **1**.

(*v*) Negative marking system is applicable for Q. No. **1** (MCQs).

MCQ

1. Choose the *correct* alternative for each of the following : 1 each

(1) If p is a polynomial such that $\deg p \geq 1$ then p has ....

(A) Exactly one root (B) At least one root

(C) At most one root (D) No root

(2) The solution ϕ of differential equation $y' = ky$, where k is a constant is given by ....

(A) $\phi(x) = e^x$ (B) $\phi(x) = e^{-x}$

(C) $\phi(x) = e^{kx}$ (D) $\phi(x) = e^{-kx}$

P.T.O.

(3) The solution ϕ of differential equation $y' + y = e^x$ is given by ....

(A) $\phi(x) = ce^x$ (B) $\phi(x) = ce^{-x}$

(C) $\phi(x) = ce^{kx}$ (D) None of these

(4) All equations $L(y) = b(x)$ are said to be non-homogeneous if ....

(A) $b(x) \neq 0$, for some x in I

(B) $b(x) = 0$, for some x in I

(C) $b(x) = 0$, for all x in I

(D) None of the above

(5) Which of the following is true ?

(A) $\| \phi(x) \| = [| \phi(x)|^2 + | \phi'(x)|^2]$

(B) $\| \phi(x) \| = [| \phi(x)| + | \phi'(x)|]^{1/2}$

(C) $\| \phi(x) \| = [| \phi(x)|^2 + | \phi'(x)|^2]^2$

(D) $\| \phi(x) \| = [| \phi(x)|^2 + | \phi'(x)|^2]^{1/2}$

(6) Two functions $\phi_1(x)$ and $\phi_2(x)$ defined on an interval I are said to be linearly dependent if ....

(A) There exist two constants c_1, c_2 not both zero s.t. $c_1\phi_1(x) + c_2\phi_2(x) = 0$, for all x in I

(B) There exist two constants c_1, c_2 at least one of which is zero s.t. $c_1\phi_1(x) + c_2\phi_2(x) = 0$, for all x in I

(C) There exist two constants c_1, c_2 both are zero s.t. $c_1\phi_1(x) + c_2\phi_2(x) = 0$, for all x in I

(D) None of the above

(7) If ϕ_1 and ϕ_2 are two solutions of $L(y) = 0$ then their Wronskian is given by ....

(A) $W(\phi_1, \phi_2) = \begin{vmatrix} \phi_1 & \phi_2 \\ \phi_1' & \phi_2' \end{vmatrix}$

(B) $W(\phi_1, \phi_2) = \phi_1 \phi_2^1 - \phi_1^1 \phi_2$

(C) Both (A) and (B) are true

(D) Both (A) and (B) are false

(8) The solution ϕ of $xy' + y = 0$ s.t. $y(1) = 1$ is given by :

(A) $\phi(x) = x$ (B) $\phi(x) = \frac{1}{x}$

(C) $\phi(x) = \frac{1}{x^2}$ (D) $\phi(x) = \frac{-1}{x}$

(9) If ϕ_1, ϕ_2 belong to the set and c_1, c_2 are any two constants, then the set of $c_1\phi_1 + c_2\phi_2$ belongs to the set is called ....

(A) Dimension (B) Basis

(C) Linear space (D) Fundamental set

(10) If

$$\phi_1(x) = \sin x \quad \text{and} \quad \phi_2(x) = \cos x$$

then $W(\phi_1, \phi_2)(x) =$ ....

(A) 1 (B) –1

(C) 0 (D) 2

P.T.O.

Theory

2. Attempt any *two* of the following : 5 each

(*a*) Consider the equation :

$$y' + ay = b(x)$$

where a is a constant, and b is a continuous function on an interval I. If x_0 is a point in I and c is any constant, then prove that the function ϕ defined by :

$$\phi(x) = e^{-ax}\int_{x_0}^{x} e^{at}b(t)\,dt + ce^{-ax}$$

is a solution of this equation.

(*b*) Consider the equation :

$$y'' = 3x + 1,$$

(*i*) Find all solutions on the interval $0 \le x \le 1$.

(*ii*) Find that solution ϕ which satisfies $\phi(0) = 1,\ \phi'(0) = 2$.

(*c*) Find all solutions of the following equation :

$$y' + 2xy = x.$$

3. Attempt any *two* of the following : 5 each

(*a*) For any real x_0 and constants α, β, there exists a solution ϕ of the initial value problem :

$$L(y) = y'' + a_1 y' + a_2 y = 0$$

$$y(x_0) = \alpha,\quad y'(x_0) = \beta$$

on $-\infty < x < \infty$. Prove this result.

(*b*) If ϕ_1, ϕ_2 are two solutions of :

$$L(y) = y'' + a_1 y' + a_2 y = 0$$

on an interval I containing a point x_0, then prove that :

$$W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)}\, W(\phi_1, \phi_2)(x_0).$$

(*c*) Find all solutions of the equation :

$$y'' + 4y = \cos x.$$

4. Attempt any *two* of the following : 5 each

(*a*) Show that if $\phi_1, \ldots\ldots., \phi_n$ are n solutions of :

$$L(y) = y^{(n)} + a_1^{(x)} y^{(n-1)} + \ldots\ldots + a_n(x) y = 0$$

on an interval I, they are linearly independent if and only if :

$$W(\phi_1, \ldots\ldots, \phi_n)(x) \neq 0$$

for all x in I.

(*b*) Find two linearly independent solutions of the equation :

$$(3x-1)^2 \; y'' + (9x-3)y' - 9y = 0$$

for $x > \frac{1}{3}$.

(*c*) Consider the equation :

$$L(y) = y'' + a,(x)y' + a_2(x)y = 0$$

where a_1, a_2 are continuous on some interval I. Let ϕ_1, ϕ_2 and Ψ_1, Ψ_2 be two bases for the solution of L(y) = 0. Show that there is a non-zero constant K such that :

$$W(\Psi_1, \Psi_2)(x) = KW(\phi_1, \phi_2)(x).$$