This question paper contains **5** printed pages]

R—72—2017

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) EXAMINATION

MARCH/APRIL, 2017

MATHEMATICS

Paper VI

(Real Analysis—I)

(MCQ + Theory)

(Saturday, 1-4-2017) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) First 30 minutes for Q. No. **1** and remaining time for other questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use black ball pen to darken the circle on OMR sheet for Q. No. **1**.

(*iv*) Negative marking system is applicable for Q. No. **1** (MCQ).

MCQ

1. Choose the most *correct* alternative of the following : 10

(1) If

$$f(x) = 1 + \sin x \ (-\infty < x < \infty) \text{ and}$$
$$g(x) = x^2 \ (0 \leq x < \infty),$$

then $g \circ f(x)$ = ...

(*a*) $1 - 2 \sin x - \sin^2 x$ (*b*) $1 - 2 \sin x + \sin^2 x$

(*c*) $1 + 2 \sin x + \sin^2 x$ (*d*) $1 + 2 \sin x - \sin^2 x$

(2) The binary expansion of $\frac{1}{16}$ is ....

(*a*) 0.1000 ...... (*b*) 0.01000 .....

(*c*) 0.00100 ..... (*d*) 0.000100 .....

P.T.O.

(3) Which of the following statements is true ?

(*a*) The set of all rational numbers is countable

(*b*) The set of all real numbers is uncountable

(*c*) The set of integers is countable

(*d*) All of the above

(4) The g.l.b. and l.u.b. of the set (7, 8) respectively are ....

(*a*) 7 and 8 (*b*) 8 and 7

(*c*) 7 and 7 (*d*) 8 and 8

(5) If

$$C = \{C_n\}_{n=1}^{\infty} = \{\sqrt{n}\}_{n=1}^{\infty} \text{ and}$$

$$N = \{n_i\}_{n=1}^{\infty} = \{i^4\}_{n=1}^{\infty}$$

then C.N. = ...

(*a*) $\{i^3\}_{n=1}^{\infty}$ (*b*) $\{i^2\}_{n=1}^{\infty}$

(*c*) $\{i\}_{n=1}^{\infty}$ (*d*) $\{-i\}_{i=1}^{\infty}$

(6) If

$$\left|\frac{2n}{n + 4n^{\frac{1}{4}}} - 2\right| < \in \qquad (n \geq N)$$

then N = ...

(*a*) $64/\in^2$ (*b*) $16/\in^2$

(*c*) $1/\in^2$ (*d*) $1/\in$

(7) If $\{S_n\}_{n=1}^{\infty}$ diverges to infinity then $\left\{(-1)^n S_n\right\}_{n=1}^{\infty}$ ....

(*a*) Converges to finite value L $\neq$ 0

(*b*) Converges to 0

(*c*) Diverges

(*d*) Oscillates

(8) If $0 < x < 1$, then $\sum_{n=1}^{\infty} x^n$ converges to ....

(*a*) $\frac{1}{1-x}$ (*b*) $\frac{1}{1+x}$

(*c*) $\frac{1}{x-1}$ (*d*) $\frac{1}{x}$

(9) If $\sum_{n=1}^{\infty} a_n \leq \sum_{n=1}^{\infty} b_n$ and $\sum_{n=1}^{\infty} |a_n| = \infty$, then $\sum_{n=1}^{\infty} |b_n| =$ ....
.

(*a*) $-\infty$ (*b*) 0

(*c*) ∞ (*d*) finite but not zero

(10) If $\lim_{n\to\infty} \sup \sqrt[n]{|a_n|} = A$, then the series of real numbers $\sum_{n=1}^{\infty} a_n$:

(*a*) converges absolutely if A < 1

(*b*) diverges if A > 1

(*c*) if A = 1 test fails

(*d*) All of the above

Theory

2. Attempt any *two* of the following : 5 each

(*i*) If

$$f : A \to B \text{ and } X \subset A,\ Y \subset A,$$

then prove that :

$$f(X \cup Y) = f(X) \cup f(Y).$$

(*ii*) If B is an infinite subset of the countable set A, then prove that B is countable.

(*iii*) If $A \subset S$ and χ_A denote the characteristic function of A, then prove that :

$$\chi_{A \cup B} = \max(\chi_A, \chi_B)$$

where A and B are subsets of S.

3. Attempt any *two* of the following : 5 each

(*i*) If

$$\{S_n\}_{n=1}^{\infty} \text{ and } \{t_n\}_{n=1}^{\infty}$$

are sequences of real numbers. If

$$\lim_{n \to \infty} S_n = L \text{ and } \lim_{n \to \infty} t_n = M$$

then prove that :

$$\lim_{n \to \infty} (S_n + t_n) = L + M$$

i.e. the limit of sum of two convergent sequences is the sum of the limits.

(*ii*) If

$$\{S_n\}_{n=1}^{\infty}$$

is a Cauchy sequence of real numbers, then prove that :

$$\{S_n\}_{n=1}^{\infty}$$

is convergent.

(*iii*) Prove that

$$\left\{\sqrt{n+1}-\sqrt{n}\right\}_{n=1}^{\infty}$$

is convergent.

4. Attempt any *two* of the following : 5 each

(*i*) Prove that the series

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

is divergent.

(*ii*) If $\sum_{n=1}^{\infty} a_n$ is a series of non-negative numbers which converges to $A \in R$ and $\sum_{n=1}^{\infty} b_n$ is a rearrangement of $\sum_{n=1}^{\infty} a_n$, then prove that $\sum_{n=1}^{\infty} b_n$ converges and $\sum_{n=1}^{\infty} b_n = A$.

(*iii*) Show that

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

converges.