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R—85—2017

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) EXAMINATION

MARCH/APRIL, 2017

MATHEMATICS

Paper VII

(Group Theory)

(MCQ & Theory)

(Wednesday, 5-4-2017) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) First 30 minutes for Q. No. **1** and remaining time for other questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use black ball pen to darken the circle on OMR sheet for Q. No. **1**.

(*iv*) Negative marking system is applicable for Q. No. **1** (MCQ).

MCQ

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) If a and b are two integers, not both zero, and if (a, b) denotes *gcd* of a and b, then which of the following is true ?

(*a*) $(a, b) = (-a, b)$ (*b*) $(a, b) = (a, -b)$

(*c*) $(a, b) = (-a, -b)$ (*d*) All of these

P.T.O.

(*ii*) Let S be a set, T = S^*, define τ : S $\to$ T by Sτ = complement of {*s*} in S = :

(*a*) S − *s* (*b*) {*s*} − S

(*c*) S − {*s*} (*d*) S

(*iii*) For the group G of all integers 0, ±1, ±2, ... with respect to the operation of usual sum of integers and $a \in$ G, what is a^{-1} ?

(*a*) 0 (*b*) 1

(*c*) $\frac{1}{a}$ (*d*) $-a$

(*iv*) If G a group, then for all $a, b \in$ G, $(a \cdot b)^{-1}$ =

(*a*) $a^{-1} \cdot b^{-1}$ (*b*) $b^{-1} \cdot a^{-1}$

(*c*) $a \cdot b^{-1}$ (*d*) $b^{-1} \cdot a$

(*v*) If H is a subgroup of a group G and for $a, b \in$ G, $a \equiv b$ mod H, then which of the following is *true* ?

(*a*) $ab \in$ H (*b*) $ab^{-1} \in$ H

(*c*) $a^{-1}b \in$ H (*d*) $a^{-1}b^{-1} \in$ H

(*vi*) If P is a prime number and *a* is any integer, then which of the following always holds ?

(*a*) $a^p \equiv 1 \bmod p$ (*b*) $a^p \equiv 0 \bmod p$

(*c*) $a^p \equiv p \bmod p$ (*d*) $a^p \equiv a \bmod p$

(*vii*) If G is a group of finite order and $a \in$ G, then how are 0(*a*) and 0(G) related :

(*a*) 0(*a*) | 0(G)

(*b*) 0(G) | 0(*a*)

(*c*) 0(*a*) > 0(G)

(*d*) 0(*a*) is always equal to 0(G)

(*viii*) Which of the following is *not* homomorphism under usual addition ?

(*a*) $\phi(x) = x + 1$ for all $x \in G$

(*b*) $\phi(x) = e$ for all $x \in G$

(*c*) $\phi(x) = 2x$ for all $x \in G$

(*d*) $\phi(x) = x$ for all $x \in G$

(*ix*) What is an isomorphism from a group G into a group $\bar{G}$?

(*a*) only homomorphism

(*b*) onto homomorphism

(*c*) one-to-one homomorphism

(*d*) one-to-one onto mapping

(*x*) If ϕ be a homomorphism of G onto $\bar{G}$ with kernel K, then :

(*a*) $G/K = G$ (*b*) $G/K \approx G$

(*c*) $G/K \approx \bar{G}$ (*d*) $G/K = \bar{G}$

Theory

2. Attempt any *two* of the following : 5 each

(*a*) If $\sigma : S \to T$, $\tau : T \to U$ and $\mu : U \to V$, then prove that :

$$(\sigma \cdot \tau) \cdot \mu = \sigma \cdot (\tau \cdot \mu).$$

(*b*) If G is a group, then prove that :

(*i*) For every $a \in G$, $(a^{-1})^{-1} = a$.

(*ii*) Every $a \in G$ has a unique inverse in G.

(*c*) If H is a subgroup of G and N is a normal subgroup of G, then show that $H \cap N$ is a normal subgroup of H.

3. Attempt any *two* of the following : 5 each

(*a*) Prove that the relation $a \equiv b \bmod H$ is an equivalence relation.

P.T.O.

(*b*) If G is a group, N a normal subgroup of G, then prove that G/N is also a group.

(*c*) Show that every subgroup of an abelian group is normal.

4. Attempt any *two* of the following : 5 each

(*a*) If ϕ is a homomorphism of G into $\bar{G}$, then prove that :

(*i*) $\phi(e) = \bar{e}$, the unit element of $\bar{G}$.

(*ii*) $\phi(x^{-1}) = \phi(x)^{-1}$, for all $x \in G$.

(*b*) If G is a group, then prove that A(G), the set of automorphisms of G, is also a group.

(*c*) Find the orbits and cycles of the permutation :

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$$