This question paper contains **6** printed pages]

R—103—2017

FACULTY OF SCIENCE

B.Sc. (Second Year) (Third Semester) EXAMINATION

MARCH/APRIL, 2017

MATHEMATICS

Paper VIII

(Ordinary Differential Equations)

(MCQ & Theory)

(Friday, 7-4-2017) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) Attempt *All* questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Negative marking system is applicable for wrong answers of MCQ.

(*iv*) Use black ball point pen to darken circle of correct answer in OMR sheet. Circle once darkened is final.

MCQ

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) If p is a polynomial such that deg $p \geqq 1$, then p has at least one root. It is known as :

(*a*) Existence Theorem

(*b*) Uniqueness Theorem

(*c*) Fundamental Theorem of Algebra

(*d*) Lagrange's Mean Value Theorem

P.T.O.

(*ii*) Consider the system of equations :

$$iz_1 + z_2 = 1 + i$$
$$2z_1 + (2 - i)z_2 = 1.$$

What are the values of z_1 and z_2 ?

(*a*) $z_1 = i$ and $z_2 = -i$ (*b*) $z_1 = -i$ and $z_2 = i$

(*c*) $z_1 = 2i$ and $z_2 = i$ (*d*) $z_1 = -2i$ and $z_2 = i$

(*iii*) A boundary condition is a condition on the solution at :

(*a*) Two or more points

(*b*) One point

(*c*) Singular and regular points

(*d*) Pole

(*iv*) The equation $L(y) = y'' + a_1 y' + a_2 y = 0$ if it has two repeated roots r_1, r_2 of characteristic polynomial p. Then its solutions are :

(*a*) $\phi_1(x) = e^{r_1 x}$ and $\phi_2(x) = e^{-r_2 x}$

(*b*) $\phi_1(x) = e^{-r_1 x}$ and $\phi_2(x) = e^{r_2 x}$

(*c*) $\phi_1(x) = e^{r_1 x}$ and $\phi_2(x) = e^{r_2 x}$

(*d*) $\phi_1(x) = e^{r_1 x}$ and $\phi_2(x) = xe^{r_1 x}$

(*v*) Which of the following is *true* ?

(*a*) $\|\phi(x)\| = \left[|\phi(x)|^2 + |\phi'(x)|^2\right]^{\frac{1}{2}}$

(*b*) $\|\phi(x)\| = \left[|\phi(x)| - |\phi'(x)|\right]^{\frac{1}{2}}$

(*c*) $\|\phi(x)\| = \left[\phi(x)\,\phi'(x)\right]^{\frac{1}{2}}$

(*d*) $\|\phi(x)\| = \left[|\phi(x)|^2 - |\phi'(x)|^2\right]^{\frac{1}{2}}$

(*vi*) Two functions ϕ_1, ϕ_2 defined on an interval I are said to be linearly dependent on I if there exist two constants c_1, c_2 not both zero, such that :

(*a*) $c_1\phi_1(x) + c_2\phi_2(x) = 1$ (*b*) $c_1\phi_1(x) + c_2\phi_2(x) = 0$

(*c*) $c_1\phi_1(x) + c_2\phi_2(x) = \infty$ (*d*) $c_1\phi_1(x) + c_2\phi_2(x) = -1$

(*vii*) A linear differential equation of order n with variable coefficients is an equation of the form :

$$a_0(x)y^{(n)} + a_1(x)y^{(n-1)} + \dots\dots\dots + a_n(x)y = b(x)$$

where $a_0, a_1, \dots\dots\dots\dots, a_n, b$ are complex-valued functions on some real interval I. Points where $a_0(x) = 0$ are called :

(*a*) Non-singular points (*b*) Boundary points

(*c*) Regular points (*d*) Singular points

(*viii*) If the coefficients a_k of L are constants, then which of the following is correct ?

(*a*) $W(\phi_1, \phi_2, \dots\dots \phi_n)(x) = e^{a_1(x + x_0)}\, W(\phi_1, \phi_2, \dots\dots \phi_n)(x_0)$

(*b*) $W(\phi_1, \phi_2, \dots\dots \phi_n)(x) = e^{a_1(x - x_0)}\, W(\phi_1, \phi_2, \dots\dots \phi_n)(x_0)$

(*c*) $W(\phi_1, \phi_2, \dots\dots \phi_n)(x) = e^{-a_1(x - x_0)}\, W(\phi_1, \phi_2, \dots\dots \phi_n)(x_0)$

(*d*) $W(\phi_1, \phi_2, \dots\dots \phi_n)(x) = e^{-a_1(x + x_0)}\, W(\phi_1, \phi_2, \dots\dots \phi_n)(x_0)$

(*ix*) The equation $L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \dots\dots\dots + a_n(x)y = b(x)$ If $b(x) = 0$ for all x on I, then $L(y) = 0$. It is known as :

(*a*) Homogeneous equation

(*b*) Non-homogeneous equation

(*c*) Ricatti equation

(*d*) Bernoulli's equation

P.T.O.

(x) The solution of the equation $y' - 2y = 1$ is :

(a) $\phi(x) = \frac{1}{2} + ce^{-2x}$, where c is any constant

(b) $\phi(x) = -\frac{1}{2} + ce^{2x}$, where c is any constant

(c) $\phi(x) = \frac{1}{2} - ce^{2x}$, where c is any constant

(d) $\phi(x) = -\frac{1}{2} - ce^{-2x}$, where c is any constant

Theory

2. Attempt any *two* of the following : 5 each

(a) Consider the equation $y' + ay = 0$ where a is a complex constant. If c is any complex number then show that the function ϕ defined by :

$$\phi(x) = ce^{-ax}$$

is a solution of this equation and moreover every solution has this form.

(b) Consider the equation $y'' = 3x + 1$:

(i) Find all solutions on the interval $0 \leqq x \leqq 1$.

(ii) Find that the solutions which satisfies $\phi(0) = 1$ and $\phi'(0) = 2$.

(c) Solve the following system for z_1, z_2, z_3 :

$$3z_1 + z_2 - z_3 = 0$$
$$2z_1 - z_3 = 1$$
$$z_2 + 2z_3 = 2.$$

3. Attempt any *two* of the following : 5 each

(a) Two solutions ϕ_1, ϕ_2 of $L(y) = 0$ are linearly independent on an interval I if and only if $W(\phi_1, \phi_2)(x) \neq 0$ for all x in I.

(*b*) Find the all solutions of the equation :

$$y'' + 9y = \sin 3x.$$

(*c*) Let a_1, a_2 be constants and consider the equation :

$$L(y) = y'' + a_2 y' + a_2 y = 0.$$

If r_1, r_2 are distinct roots of the characteristic polynomial p, where $p(r) = r^2 + a_1 r + a_2$ then prove that the functions ϕ_1, ϕ_2 defined by :

$$\phi_1(x) = e^{r_1 x}, \ \phi_2(x) = e^{r_2 x}$$

are solution of $L(y) = 0$.

4. Attempt any *two* of the following : 5 each

(*a*) Let $b_1, b_2, \ldots\ldots, b_n$ be non-negative constants such that for all x in I :

$$|a_j(x)| \le b_j, \ (j = 1, 2, \ldots\ldots, n)$$

and define K by

$$K = 1 + b_1 + b_2 + \ldots\ldots + b_n.$$

If x_0 is a point on I and ϕ is a solution of $L(y) = 0$ on I then prove that :

$$\left\|\phi(x_0)\right\| e^{-K|x - x_0|} \leqq \left\|\phi(x)\right\| \leqq \left\|\phi(x_0)\right\| e^{K|x - x_0|}$$

for all x in I.

(*b*) Find two linearly independent solutions of the equation :

$$(3x - 1)^2 y'' + (9x - 3) y' - 9y = 0 \text{ for } x > \frac{1}{3}.$$

P.T.O.

(*c*) Consider the equation $L(y) = y'' + a_1 y' + a_2 y = 0$ show that a_1 and a_2 are uniquely determined by any basis ϕ_1, ϕ_2 for the solutions of $L(y) = 0$ it satisfying $L(\phi_1) = L(\phi_2) = 0$ show that :

$$a_1 = \frac{-\begin{vmatrix} \phi_1 & \phi_2 \\ \phi_1'' & \phi_2'' \end{vmatrix}}{W(\phi_1, \phi_2)} \text{ and } a_2 = \frac{\begin{vmatrix} \phi_1' & \phi_2' \\ \phi_1'' & \phi_2'' \end{vmatrix}}{W(\phi_1, \phi_2)}.$$