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V—69—2017

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) EXAMINATION

OCTOBER/NOVEMBER, 2017

(CBCS/CGPA Pattern)

MATHEMATICS

Paper VI

(Real Analysis—I)

(MCQ & Theory)

(Saturday, 11-11-2017) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes are for Q. No. **1** and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball pen to darken the circle on OMR sheet for Q. No. **1**.

(*v*) Negative marking system is applicable for Q. No. **1**.

MCQ

1. Choose the most *correct* alternative for each of the following : 1 each

(*i*) Which of the following is countable set ?

(*a*) The set of all irrational numbers

(*b*) The set of all real numbers

(*c*) $[0, 1] = \{x : 0 \le x \le 1\}$

(*d*) Set of all rational numbers

P.T.O.

(*ii*) The binary expansion of $\frac{1}{16}$ is :

(*a*) 0.00100........ (*b*) 0.01000........

(*c*) 0.000100........ (*d*) 0.1000........

(*iii*) Which of the following does *not* hold ?

(*a*) $\chi_{A \cup B}(x) = \max(\chi_A, \chi_B)$

(*b*) $\chi_{A - B}(x) = \chi_A + \chi_B$

(*c*) $\chi_{A \cap B}(x) = \min(\chi_A, \chi_B)$

(*d*) $\chi_{A'}(x) = 1 - \chi_A(x)$

(*iv*) If $\{S_n\}_{n=1}^{\infty}$ is a convergent sequence of real numbers, then :

(*a*) $\lim_{n\to\infty} \sup S_n \geq \lim_{n\to\infty} S_n$ (*b*) $\lim_{n\to\infty} \sup S_n = \lim_{n\to\infty} S_n$

(*c*) $\lim_{n\to\infty} \sup S_n \leq \lim_{n\to\infty} S_n$ (*d*) None of these

(*v*) $\lim_{n\to\infty} \frac{3n^2 - 6n}{5n^2 + 4} =$ ....

(*a*) $\frac{5}{3}$ (*b*) 5

(*c*) 3 (*d*) $\frac{3}{5}$

(*vi*) If $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers, then $\{S_n\}_{n=1}^{\infty}$ is :

(*a*) Bounded

(*b*) Convergent

(*c*) Bounded and Convergent

(*d*) Divergent

(*vii*) If $\left| \frac{2n}{n + 4n^{1/4}} - 2 \right| < \in \ (n \geq N)$, then N = ...

(*a*) $64/\in^2$ (*b*) $16/\in^2$

(*c*) $1/\in^2$ (*d*) $1/\in$

(*viii*) If $0 < x < 1$, then $\sum_{n=1}^{\infty} x^n$ converges to :

(*a*) $\frac{x^n}{1-x}$ (*b*) $\frac{1}{1-x}$

(*c*) $\frac{1}{x-1}$ (*d*) $\frac{1}{x+1}$

(*ix*) An infinite series is convergent if its sequence of partial sum :

(*a*) Oscillates (*b*) is Bounded

(*c*) is Convergent (*d*) is Divergent

(*x*) If $\sum_{n=1}^{\infty} a_n \leq \sum_{n=1}^{\infty} b_n$ and $\sum_{n=1}^{\infty} |b_n| < \infty$, then :

(*a*) $\sum_{n=1}^{\infty} |a_n| < \infty$ (*b*) $\sum_{n=1}^{\infty} |a_n| > \infty$

(*c*) $\sum_{n=1}^{\infty} |a_n| = \infty$ (*d*) None of these

Theory

2. Attempt any *two* of the following : 5 each

(*a*) If $F : A \to B$ and $X \subset B$, $Y \subset B$, then prove that :

$$F^{-1}(X \cup Y) = F^{-1}(X) \cup F^{-1}(Y).$$

(*b*) If $A_1, A_2, A_3, \ldots\ldots\ldots\ldots$ are countable sets, then prove that $\bigcup_{n=1}^{\infty} A_n$ is countable.

(*c*) If A, B are subsets of S, then prove that :

$$\chi_{A \cap B}(x) = \min(\chi_A, \chi_B).$$

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ converges to L, then prove that $\{S_n\}_{n=1}^{\infty}$ cannot also converges to a limit distinct from L. i.e. if $\lim_{n\to\infty} S_n = L$ and $\lim_{n\to\infty} S_n = M$, then L = M.

(*b*) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ converges, then prove that $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

(*c*) Prove that $\left\{\sqrt{n+1} - \sqrt{n}\right\}_{n=1}^{\infty}$ is convergent.

4. Attempt any *two* of the following : 5 each

(*a*) If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then prove that $\sum_{n=1}^{\infty} a_n$ converges.

(*b*) Let $\sum_{n=1}^{\infty} a_n$ be a series of non-zero real numbers and let

$a = \liminf_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$, $A = \limsup_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$, then prove that if $A < 1$, then $\sum_{n=1}^{\infty} |a_n| < \infty$.

(*c*) Prove that the series $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)$ is divergent.