This question paper contains **5** printed pages]

# V—98—2017

## FACULTY OF SCIENCE

**B.Sc. (Second Year) (Third Semester) EXAMINATION**

**OCTOBER/NOVEMBER, 2017**

**(CBCS/CGPA)**

MATHEMATICS

Paper VIII

(Ordinary Differential Equations)

(MCQ+Theory)

**(Thursday, 16-11-2017)** **Time : 2.00 p.m. to 4.00 p.m.**

*Time—Two Hours* *Maximum Marks*—40

*N.B.* :— (*i*) First 30 minutes for Q.No. **1** and remaining time for other questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use black ball pen to darken the circle on OMR sheet for Q. No. **1**.

(*iv*) Negative marking system is applicable for Q. No. **1** (MCQs)

**MCQ**

1. Select the *correct* answer :

(*i*) If $p$ is polynomial deg $p \geqq 1$ with leading coefficient $a_0 \neq 0$, then $p$ has :

(A) At least one root

(B) Exactly $n$ roots

(C) One or two roots

(D) Infinitely many roots

(*ii*) Let $p$ be a polynomial of deg $n \geqq 1$ with leading coefficient 1 and let $r$ be a root of $p$. Then $p(z) = (z - r)\, q(z)$ where :

(A) $q$ is a polynomial of degree $n + 1$ with leading coefficient 1

(B) $q$ is a polynomial of degree $n - 1$ with leading coefficient 1

(C) $q$ is a polynomial of degree $n + 2$ with leading coefficient 1

(D) $q$ is a polynomial of degree $n - 2$ with leading coefficient 1

P.T.O.

(*iii*) An initial condition is a condition on the solution at :

(A) two points

(B) two or more than two points

(C) infinite points

(D) one point

(*iv*) The characteristic polynomial of $y'' - y' + 6y = 0$ is :

(A) $r^2 + r + 6$

(B) $r^2 - r + 6$

(C) $r^2 + 6$

(D) $r - 6$

(*v*) The solution of $y' + 3y = \cos x$ is :

(A) $\phi(x) = \frac{1}{10}\left[3\cos x + \sin x\right] + Ce^{-3x}$

(B) $\phi(x) = \frac{-1}{10}\left[3\cos x + \sin x\right] + Ce^{3x}$

(C) $\phi(x) = \frac{1}{10}\left[3\cos x - \sin x\right] + Ce^{3x}$

(D) $\phi(x) = \frac{-1}{10}\left[3\cos x - \sin x\right] + Ce^{3x}$

(*vi*) The solution of $y' + y = e^x$ is :

(A) $\phi(x) = \frac{e^{-\lambda}}{2} + Ce^{\lambda}$ where C is any constant

(B) $\phi(x) = \frac{e^{x}}{2} + Ce^{-x}$ where C is any constant

(C) $\phi(x) = \frac{-e^{-\lambda}}{2} + Ce^{\lambda}$

(D) $\phi(x) = \frac{e^{-\lambda}}{3} + Ce^{-x}$

(*vii*) If $L(y) = y^{(n)} + a_1 y^{(n-1)} + \ldots\ldots\ldots + a_n y = b(x)$

If $b(x) \neq 0$ for some $x \in I$, then $L(y) = b(x)$ is called :

(A) Riccati equation

(B) Homogeneous equation

(C) Non-homogeneous equation

(D) Lagrange's equation

(*viii*) Let $\alpha, \beta$ be any two constants and $x_0$ be any real number. On any interval I containing $x_0$ there exists at most one solution $\phi$ of the initial value problem :

$$L(y) = 0,\ y(x_0) = \alpha,\ y'(x_0) = \beta$$

It is known as :

(A) Uniqueness theorem (B) Format theorem

(C) Existence theorem (D) Cauchy theorem

(*ix*) The solution of the equation $y'' + \frac{1}{x} y' - \frac{1}{x^2} y = 0$ is :

(A) $\frac{1}{x^2}$ (B) $\frac{1}{x}$

(C) $\frac{1}{x^3}$ (D) $\frac{1}{x^4}$

(*x*) If $b_1, b_2, \ldots\ldots\ldots\ldots b_n$ be non-negative constants such that for all $x$ in I $|a_j(x)| \leq b_j$ $(j = 1, 2, \ldots\ldots\ldots n)$ defined by $k = 1 + b_1 + b_2 + \ldots\ldots\ldots\ldots + b_n$. If $\lambda_0$ is a point in I and $\phi$ is a solution of $L(y) = y^{(n)} + a_1(x)\, y^{(n-1)} + \ldots\ldots\ldots\ldots\ldots + a_n(x) y = 0$ on I, then :

(A) $\| \phi(x) \| \, e^{-k|x + x_0|} \leqq \| \phi(x) \| \leqq \| \phi(x_1) \| \, e^{-k|x - x_0|}$

(B) $\| \phi(x_0) \| \, e^{-k|x - x_0|} \leqq \| \phi(x) \| \leqq \| \phi(x_0) \| \, e^{k|x - x_0|}$

(C) $\| \phi(x_0) \| \, e^{k|x - x_0|} \leqq \| \phi(x) \| \leqq \| \phi(x_0) \| \, e^{-k|x - x_0|}$

(D) $\| \phi(x) \| \, e^{k|x - x_0|} \leqq \| \phi(x_0) \| \leqq \| \phi(x) \| \, e^{-k|x - x_0|}$

## Theory

2. Attempt any *two* of the following : 5 each

(*a*) If $r$ is a root of multiplicity $m$ of a polynomial P, deg $p \geqq 1$, then prove that :

$$p(r) = p'(r) = \dots\dots\dots\dots p_{(r)}^{(m-1)} = 0$$

and $p^m(r) \neq 0$.

(*b*) Consider the equation $y' + ay = b(x)$ where $a$ is constant and $b$ is continuous function on an interval I. If $x_0$ is a point in I and C is any constant prove that the function $\phi$ defined :

$$\phi(x) = e^{-ax} \int_{x_0}^{x} e^{at}\, b(t)\, dt + ce^{-ax}$$

is a solution of this equation.

(*c*) Prove that $\phi(x) = e^{-\sin x}$ is a solution of differential equation :

$$y' + (\cos x)y = 0.$$

3. Attempt any *two* of the following : 5 each

(*a*) If $\phi_1, \phi_2$ are solution of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing a point $x_0$, then prove that :

$$W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)}\, W(\phi_1, \phi_2)(x_0).$$

(*b*) For any real $x_0$ and constants $\alpha$, $\beta$, prove that there exists a solution $\phi$ of initial value problem $L(y) = 0$ $y(x_0) = \alpha$, $y'(x_0) = \beta$ on $-\infty < x < \infty$.

(*c*) Find all solutions of the equation :

$$y'' + y - 2y = 0.$$

4. Attempt any *two* of the following : 5 each

(*a*) Let $\phi_1, \phi_2 \dots\dots\dots\dots \phi_n$ be $n$ linearly independent solution of :

$$L(y) = y^{(n)} + a_1(x)\, y^{(n-1)} + \dots\dots\dots\dots + a_n(x)\, y = 0$$

on an interval I. If $\phi$ is any solution of $L(y) = 0$ on I, then prove that $\phi$ can be represented in the form :

$$\phi = C_1\phi_1 + C_2\phi_2 + \ldots\ldots\ldots + C_n\phi_n$$

where $C_1, C_2, \ldots\ldots\ldots C_n$ are constants.

(*b*) Find two linearly independent solution of the equation :

$$(3x-1)^2 y'' + (9x-3)y' - 9y = 0 \text{ for } x > \frac{1}{3}.$$

(*c*) Consider the equation $y'' + \alpha(x)y = 0$ where $\alpha$ is a continuous function on $-\infty < x < \infty$ which is period $\xi > 0$, let $\phi_1, \phi_2$ be basis for the solutions satisfying :

$$\phi_1(0) = 1, \quad \phi_2(0) = 0$$

$$\phi_1'(0) = 0, \quad \phi_2'(0) = 1$$

show that $W(\phi_1, \phi_2)(x) = 1$ for all $x$.