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AO—68—2018

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) EXAMINATION

MARCH/APRIL, 2018

(CBCS/CGPA Pattern)

MATHEMATICS

Paper VI

(Real Analysis—I)

(MCQ & Theory)

(Monday, 26-3-2018) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 min. for MCQ i.e. Q. No. **1** and remaining time for Theory i.e. for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball pen to darken the circles on OMR.

(*v*) Negative marking system is applicable for Question No. **1**.

MCQ

1. Choose most *correct* alternative for each of the following : 1 each

(*i*) If $f(x) = \sin x$, $(-\pi/2 \le x \le \pi/2)$, then $f^{-1}(1) =$ ....

(*a*) $\pi/2$ (*b*) π

(*c*) 2π (*d*) $+3\pi/2$

P.T.O.

(*ii*) If f and g are two real-valued functions defined on R, then $\min(f, g) =$ ....

(*a*) $\frac{|f - g| + f - b}{2}$ (*b*) $\frac{|f - g| - f - g}{2}$

(*c*) $\frac{-|f - g| - g - b}{2}$ (*d*) $\frac{-|f - g| + f + g}{2}$

(*iii*) Which of the following does *not* hold ?

(*a*) $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$

(*b*) $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$

(*c*) $f(X \cap Y) = f(X) \cap f(Y)$

(*d*) $f(X \cup Y) = f(X) \cup f(Y)$

(*iv*) The binary expansion of 1/2 is :

(*a*) 0.10000 (*b*) 0.01000

(*c*) 0.00010 (*d*) 0.11010

(*v*) If $S_n = \frac{10^n}{n!}$, then the value of $N \in I$ such that $S_{n+1} < S_n$, $(n \geq N)$, is :

(*a*) 9 (*b*) 10

(*c*) 100 (*d*) 99

(*vi*) If $C = \{C_n\}_{n=1}^{\infty} = \{\sqrt{n}\}_{n=1}^{\infty}$ and $N = \{n_i\}_{i=1}^{\infty} = \{i^4\}_{i=1}^{\infty}$, then $C \circ N =$

(*a*) $\{i^4\}_{i=1}^{\infty}$ (*b*) $\{i^2\}_{i=1}^{\infty}$

(*c*) $\{i\}_{i=1}^{\infty}$ (*d*) $\{i^6\}_{i=1}^{\infty}$

(*vii*) If $\{S_n\}_{n=1}^{\infty}$ diverges to infinity, then $\{(-1)^n S_n\}_{n=1}^{\infty}$:

(*a*) Converges to finite value $L \neq 0$

(*b*) Converges to zero

(*c*) Diverges

(*d*) Oscillates

(*viii*) If the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is convergent, then the partial sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is :

(*a*) 1 (*b*) $1 - \frac{1}{n+1}$

(*c*) $1 + \frac{1}{n+1}$ (*d*) $\frac{1}{n+1}$

(*ix*) Let $\sum_{n=1}^{\infty} a_n$ be a series on non-zero real numbers and let :

$$a = \liminf_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|, \ A = \limsup_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|,$$

then which of the following are possible ?

(*a*) if $A < 1$, then $\sum_{n=1}^{\infty} |a_n| < \infty$

(*b*) If $a > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges

(*c*) $1 \leq 1 \leq A$, then test fails

(*d*) All of the above

P.T.O.

(*x*) If $\sum_{n=1}^{\infty} a_n$ converges absolutely to A, then any rearrangement $\sum_{n=1}^{\infty} b_n$ of $\sum_{n=1}^{\infty} a_n$ is :

(*a*) Converges to infinite (*b*) Converges to A

(*c*) Converges absolutely to A (*d*) Converges absolutely to zero

Theory

2. Attempt any *two* of the following : 5 each

(*a*) Prove that the set of all rational numbers is countable.

(*b*) If A is any non-empty subset of R which is bounded below, then show that A has greatest lower bound in R.

(*c*) Consider the function '*f*' defined by $f(x) = \tan x$ $(-\pi/2 < x < \pi/2)$:

(*i*) Find domain and range of '*f*'.

(*ii*) If $A = (-\pi/2, -\pi/4)$, $B = (\pi/4, \pi/2)$, then find $f(A)$, $f(B)$, $f(A \cap B)$.

(*iii*) Does $f(A \cap B) = f(A) \cap f(B)$ hold in this case ? Justify.

3. Attempt any *two* of the following : 5 each

(*a*) Show that a non-decreasing sequence which is bounded above is convergent.

(*b*) If $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers, then prove that $\{S_n\}_{n=1}^{\infty}$ is bounded.

(*c*) Prove that $\{S_n\}_{n=1}^{\infty} = \left\{\frac{2n}{n+4n^{1/2}}\right\}_{n=1}^{\infty}$ and $\{S_n\}_{n=1}^{\infty} = \left\{\frac{2n}{n+3}\right\}$, have a limit 2.

4. Attempt any *two* of the following : 5 each

(*a*) Show that if $\sum_{n=1}^{\infty} a_n$ is convergent series, then $\lim_{n \to \infty} a_n = 0$.

(*b*) Prove that $1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{5!} + ...$ converges.

(*c*) If $\limsup_{n \to \infty} \sqrt[n]{|a_n|} = A$, then show that the series of real numbers $\sum_{n=1}^{\infty} a_n$ converges absolutely if $A < 1$ and diverges if $A > 1$.