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AO—80—2018

FACULTY OF ARTS AND SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) EXAMINATION

MARCH/APRIL, 2018

(CBCS/CGPA)

MATHEMATICS

Paper VII

(Group Theory)

(MCQ & Theory)

(Wednesday, 28-03-2018) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) First 30 minutes for Q. No. **1** and remaining time for other questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use black ball pen to darken the circle on OMR sheet for Q. No. **1**.

(*iv*) Negative marking system is applicable for Q. No. **1** (MCQs).

MCQ

1. Choose the correct alternative for each of the following : 1 each

(*i*) The mapping τ of S into T is said to be onto T if given $t \in$ T there exists an element $s \in$ S such that :

(*a*) $\tau = st$ (*b*) $t = s\tau$

(*c*) $s = t\tau$ (*d*) None of these

(*ii*) Greatest common divisor of (60, 24) :

(*a*) –12 (*b*) –24

(*c*) 12 (*d*) 24

(*iii*) If a and b are integers, not both 0, then (a, b) exists, moreover, we can find integers m_0 and n_0 such that $(a, b) =$

(*a*) $ma + nb$ (*b*) $m + n$

(*c*) $a + b$ (*d*) $moa + nob$

P.T.O.

(*iv*) If G is a group and $a, u, w \in G$, then cancellation law is :

(*a*) $a \cdot u = a \cdot w$ implies $u = w$ (*b*) $u \cdot a = a \cdot u$

(*c*) $u \cdot (a \cdot w) = (u \cdot a) w$ (*d*) None of these

(*v*) If H is a subgroup of G, $a \in G$, then which of the following is a right coset of H in G ?

(*a*) $Ha = \{ah \mid h \in H\}$ (*b*) $Ha = \{ha \mid h \in H\}$

(*c*) $Ha = \{a^{-1}ha \mid h \in H\}$ (*d*) $Ha = \{ah^{-1}a \mid h \in H\}$

(*vi*) If p is a prime number and a is any integer, then $a^p \equiv d \bmod p$. This theorem is known as :

(*a*) Euler theorem (*b*) Lagrange's theorem

(*c*) Sylow's theorem (*d*) Fermat theorem

(*vii*) If G is a finite group and N is a normal subgroup of G, then $o(G/N) =$

(*a*) $o(G) + o(N)$ (*b*) $o(G) - o(N)$

(*c*) $o(G)/o(N)$ (*d*) $o(G) \times o(N)$

(*viii*) A mapping φ from a group G into a group $\overline{G}$ is said to be a homomorphism if for all $a, b \in G$:

(*a*) $\varphi(ab) = a\varphi(b)$ (*b*) $\varphi(ab) = \varphi(a)\ \varphi(b)$

(*c*) $\varphi(ab) = b\varphi(a)$ (*d*) $ab = (a)\ (b)$

(*ix*) Let G, G* and G* be the groups. Then which of the following is most correct ?

(*a*) G is isomorphic to itself

(*b*) G is isomorphic to G*, then G* is isomorphic to G

(*c*) G is isomorphic to G*, G* is isomorphic to G**, then G is isomorphic to G**

(*d*) All of the above

(*x*) Every permutation is a product of :

(*a*) 2 – cycles (*b*) 3 – cycles

(*c*) 4 – cycles (*d*) 5 – cycles

Theory

2. Attempt any *two* of the following : 5 each

(*a*) If a is relatively prime to b but $a | bc$, then prove that $a | c$.

(*b*) If G is a group, then prove that :

(*i*) Every $a \in G$ has a unique inverse in G.

(*ii*) For all $a, b \in G$, $(a \cdot b)^{-1} = b^{-1} \cdot a^{-1}$.

(*c*) Prove that if G is an abelian group, then for all $a, b \in G$ and all integers n,

$$(a \cdot b)^n = a^n \cdot b^n$$

3. Attempt any *two* of the following : 5 each

(*a*) Prove that a non-empty subset H of a group G is a subgroup of G if and only if,

(*i*) $a, b \in H$ implies that $ab \in H$.

(*ii*) $a \in H$ implies that $a^{-1} \in H$.

(*b*) Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

(*c*) Show that the intersection of two normal subgroups of G is a normal subgroup of G.

4. Attempt any *two* of the following : 5 each

(*a*) Suppose G is a group, N a normal subgroup of G, define the mapping φ from G to $G | N$ by $\varphi(x) = Nx$ for all $x \in G$. Then prove that φ is a homomorphism of G onto $G | N$.

(*b*) Prove that every group is isomorphic to a subgroup of A(s) for some apropriate S.

(*c*) Let G be a group, H a subgroup of G, T an automorphism of G. Let $H(T) = \{hT | h \in H\}$. Prove that (H)T is a subgroup of G.