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W—109—2018

FACULTY OF SCIENCE

B.Sc. (Second Year) (Third Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(CBCS/CGPA Pattern)

MATHEMATICS

Paper VIII

(Ordinary Differential Equations)

(MCQ & Theory)

(Tuesday, 23-10-2018) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) First 30 minutes for Q. **1** and remaining time for other questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use black ball pen to darken the circle on OMR sheet for Q. **1**.

(*iv*) Negative marking system is applicable for Q. **1** (MCQs).

MCQ

1. (*i*) If p is a polynomial such that deg $p \geq 1$, then p has at least one root. It is known as :

(*a*) Lagrange's Theorem

(*b*) Fundamental Theorem of Algebra

(*c*) Taylor's Theorem

(*d*) Cauchy's Theorem

(*ii*) It r is such that $r^3 = 1$ and $r \neq 1$ then :

(*a*) $1 + r^2 = 0$ (*b*) $1 - r + r^2 = 0$

(*c*) $1 + r + r^2 = 0$ (*d*) $1 + r - r^2 = 0$

P.T.O.

(*iii*) If system of equations

$$3z_1 + z_2 - z_3 = 0$$
$$2z_1 - z_3 = 0$$
$$z_2 + 2z_3 = 0$$

then the value of z_1, z_2, z_3 are :

(*a*) $\frac{5}{3}, \frac{-8}{3}, \frac{7}{3}$ (*b*) $\frac{-8}{3}, \frac{-5}{3}, \frac{-7}{3}$

(*c*) $\frac{8}{3}, \frac{-5}{3}, \frac{7}{3}$ (*d*) $\frac{8}{3}, \frac{8}{3}, \frac{7}{3}$

(*iv*) A binary condition is a condition on the solution at :

(*a*) Two or more points (*b*) Two and less than two points

(*c*) Infinitely many points (*d*) One point

(*v*) The solution of the equation $y' + 5y = 2$ is :

(*a*) $\phi(x) = \frac{-1}{3} + ce^{-3x}$ (*b*) $\phi(x) = \frac{1}{3} + ce^{3x}$

(*c*) $\phi(x) = \frac{-2}{5} + ce^{5x}$ (*d*) $\phi(x) = \frac{2}{5} + ce^{-5x}$

(*vi*) The equation $y' + ay = b(\lambda)$ has a solution :

(*a*) $\phi(x) = e^{ax}\,B(x) + ce^{-ax}$ (*b*) $\phi(x) = e^{-ax}\,B(x) + ce^{ax}$

(*c*) $\phi(x) = e^{-ax}\,B(x) + ce^{-ax}$ (*d*) $\phi(x) = e^{ax}\,B(x) + ce^{ax}$

(*vii*) Two solutions ϕ_1, ϕ_2 defined on an interval I are said to be linearly dependent on I if there exist two constants c_1, c_2 not both zero such that :

(*a*) $c_1\phi_1(x) + c_2\phi_2(x) = 1$ (*b*) $c_1\phi_1(x) + c_2\phi_2(x) = 0$

(*c*) $c_1\phi_1(x) + c_2\phi_2(x) = 2$ (*d*) $\frac{1}{2}c_1\phi_1(x) + \frac{1}{3}c_2\phi_2(x) = 3$

(*viii*) If ϕ_1, ϕ_2 are two solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing a point x_0, then :

(*a*) $W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)} W(\phi_1, \phi_2)(x_0)$

(*b*) $W(\phi_1, \phi_2)(x) = e^{a_1(x-x_0)} W(\phi_1, \phi_2)(x_0)$

(*c*) $W(\phi_1, \phi_2)(x) = e^{\frac{a_1(x+x_0)}{2}} W(\phi_1, \phi_2)(x_0)$

(*d*) $W(\phi_1, \phi_2)(x) = e^{\frac{-a_1(x+x_0)}{2}} W(\phi_1, \phi_2)(x_0)$

(*ix*) A linear differential equation of order n with variable coefficients is of the form $a_0(x)y^{(n)} + a_1(x)y^{(n-1)} + \ldots\ldots + a_n(x)y = b(x)$ points where $a_0(x) = 0$ are called :

(*a*) Trivial points (*b*) Regular and singular points

(*c*) Regular points (*d*) Singular points

(*x*) If $y' = -y^2 - a_1(x)y - a_2(x)$ is obtained from $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ is known as :

(*a*) Bernoulli's equation (*b*) Riccati equation

(*c*) Fermat's equation (*d*) Euler's equation

Theory

2. Attempt any *two* of the following : 5 each

(*a*) If the determinant Δ of the coefficients in :

$$\begin{aligned} a_{11}z_1 + a_{12}z_2 + \ldots\ldots\ldots\ldots + a_{1n}z_n &= c_1 \\ a_{21}z_1 + a_{22}z_2 + \ldots\ldots\ldots\ldots + a_{2n}z_n &= c_2 \\ \vdots \qquad \vdots \qquad\qquad\qquad \vdots \quad &\quad \vdots \\ a_{n1}z_1 + a_{n2}z_2 + \ldots\ldots\ldots\ldots + a_{nn}z_n &= c_n \end{aligned}$$

is not zero. Then prove that there is a unique solution of the system for $z_1, z_2, \ldots\ldots, z_n$. It is given by

$$z_k = \frac{\Delta k}{\Delta} \ (k = 1, 2, \ldots, n)$$

where Δk is determinant obtained by replacing its kth column $a_{1k}, a_{2k}, \ldots\ldots, a_{nk}$ by $c_1, c_2, \ldots\ldots\ c_n$.

P.T.O.

(*b*) Consider the equation $y' + ay = 0$ where a is complex constant. If c is any complex number, prove that the function ϕ defined by $\phi(x) = ce^{-ax}$ is a solution of this equation and moreover every solution has this form.

(*c*) Find all solutions of the equation $y' + e^x y = 3e^x$.

3. Attempt any *two* of the following : 5 each

(*a*) If a_1, a_2 be constants and consider the equation $L(y) = y'' + a_1 y' + a_2 y = 0$. If r_1, r_2 are distinct roots of the characteristic polynomial p where $p(r) = r^2 + a_1 r + a_2$, then prove that the functions ϕ_1, ϕ_2 defined by

$$\phi_1(x) = e^{r_1 x},\ \phi_2(x) = e^{r_2 x}$$

are solution of $L(y) = 0$. If r_1 is repeated root of p, then prove that the functions ϕ_1, ϕ_2 defined by

$$\phi_1(x) = e^{r_1 x},\ \phi_2(x) = xe^{r_1 x}$$

are solutions of $L(y) = 0$.

(*b*) Find the solution of initial value problem $y'' - 2y' - 3y = 0$, $y(0) = 0$, $y'(0) = 1$.

(*c*) Show that the functions ϕ_1, ϕ_2 defined by $\phi_1(x) = x^2$, $\phi_2(x) = x|x|$ are linearly independent for $-\infty < x < \infty$ and find Wronskian of these functions.

4. Attempt any *two* of the following : 5 each

(*a*) If $\phi_1, \phi_2, ..., \phi_n$ are n solutions of $L(y) = y^{(x)} + a_1(x)y^{(n-1)} + ... + a_n(x)y = 0$ on an interval I, then prove that they are linearly independent there if and only if

$$W(\phi_1, \phi_2, ..., \phi_n)(x) \neq 0 \text{ for all } x \in I.$$

(*b*) If $\phi_1, \phi_2, \ldots\ldots, \phi_n$ be n solutions of $L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \ldots\ldots + a_n y = 0$ on I satisfying $\phi_i^{(i-1)}(x_0) = 1, \phi_1^{(j-i)}(x_0) = 0, j \neq i$. Prove that if ϕ is any solution of $L(y) = 0$ on I, there are n constants $c_1, c_2, \ldots\ldots, c_n$ such that

$$\phi = c_1\phi_1 + c_2\phi_2 + \ldots\ldots + c_n\phi_n.$$

(*c*) Consider the equation

$$y'' + \frac{1}{x}y' - \frac{1}{x^2}y = 0 \quad \text{for } x > 0$$

Show that there is a solution of the form x^r, where r is a constant.