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W—77—2018

FACULTY OF SCIENCE/ARTS

B.Sc./B.A. (Second Year) (Third Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(CBCS/CGPA Pattern)

MATHEMATICS

Paper VI

(Real Analysis-I)

(MCQ+Theory)

(Wednesday, 17-10-2018) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes for Q. No. **1** and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball pen to darken the circle on OMR sheet for Q. No. **1**.

(*v*) Negative marking system is applicable for Q. No. **1** (MCQ).

(MCQ)

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) Let $f(x) = \log x$ $(0 < x < \infty)$, then the range of f is :

(A) $(0, \infty)$ (B) $(-\infty, 0)$

(C) $(-\infty, \infty)$ (D) $(0, 1)$

(*ii*) If A is any non-empty subset of R that is bounded above, then A has a .... in R.

(A) no least upper bound

(B) no greatest lower bound

(C) greatest lower bound

(D) least upper bound

P.T.O.

(*iii*) If $f(x) = 1 + \sin x (-\infty, < x < \infty)$ and $g(x) = x^2 (0, \leq x < \infty)$, then $g \text{ o } f(x) =$ ....

(A) $1 - 2 \sin x - \sin^2 x$

(B) $1 - 2 \sin x + \sin^2 x$

(C) $1 + 2 \sin x + \sin^2 x$

(D) $1 + 2 \sin x - \sin^2 x$

(*iv*) Let $\{S_n\}_{n=1}^{\infty}$ be a sequence defined by $S_1 = 1$, $S_2 = 1$

$$S_{n+1} = S_n + S_{n-1} \ (n = 3, 4, 5, \ldots\ldots\ldots)$$

then $S_8 =$ ....

(A) 21

(B) 13

(C) 34

(D) 25

(*v*) If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers which converges to L, then $\{S_n^2\}_{n=1}^{\infty}$ converges to ....

(A) L

(B) 1

(C) L^2

(D) ∞

(*vi*) The value of $N \in I$ such that $\left| \frac{2n}{n+3} - 2 \right| < \frac{1}{5} \ (n \geq N)$ holds, N =

(A) 27

(B) 15

(C) 4

(D) 20

(*vii*) Let $N = \{n_i\}_{i=1}^{\infty}$ be a sequence of positive integers where each positive integer occurs exactly one among the n_i. If $\sum_{n=1}^{\infty} a_n$ is a series of real numbers and if $b_i = a_{n_i} \ (i \in I)$, then $\sum_{i=1}^{\infty} b_i$ is called .... of $\sum_{n=1}^{\infty} a_n$.

(A) Non-negative term series

(B) Rearrangement series

(C) Non-positive term series

(D) None of the above

(*viii*) If $\sum_{n=1}^{\infty} a_n$ is dominated by $\sum_{n=1}^{\infty} b_n$ and $\sum_{n=1}^{\infty} |a_n| = \infty$, then $\sum_{n=1}^{\infty} |b_n| =$ ....

(A) $-\infty$ (B) 0

(C) ∞ (D) finite but not zero

(*ix*) If $x \geq 1$, then $\sum_{n=1}^{\infty} x^n$:

(A) diverges (B) converges

(C) oscillates (D) tends to zero

(*x*) If $\limsup_{n\to\infty} \sqrt[n]{|a_n|} = A$, then the series of real numbers $\sum_{n=1}^{\infty} a_n$ diverges if :

(A) $A < 1$ (B) $A > 1$

(C) $A = 1$ (D) $A = 0$

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) If $f : A \to B$ and $X \subset B$, $Y \subset B$, then prove that :

$$f(X \cup Y) = f(X) \cup f(Y).$$

(*b*) If A, B are subsets of S, then prove that :

$$\chi_{A \cup B}(X) = \max(\chi_A, \chi_B).$$

(*c*) Consider the sine function defined by $f(x) = \sin x, (-\infty < x < \infty)$, then :

(*i*) what is the image $\frac{\pi}{2}$ of under f

(*ii*) find $f^{-1}\left(\frac{1}{2}\right)$

(*iii*) find $f\left(\left[0, \frac{\pi}{6}\right]\right)$, $f\left(\left[\frac{\pi}{6}, \frac{\pi}{2}\right]\right)$ and $f\left(\left[0, \frac{\pi}{2}\right]\right)$.

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent, then prove that $\{S_n\}_{n=1}^{\infty}$ is bounded.

(*b*) If $\{S_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers, then prove that $\{S_n\}_{n=1}^{\infty}$ is convergent.

(*c*) For $n \in I$, let $t_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots\ldots + \frac{1}{n!}$, then prove that $\{t_n\}_{n=1}^{\infty}$ is non-decreasing and bounded.

4. Attempt any *two* of the following : 5 each

(*a*) If $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive numbers such that :

(*i*) $a_1 \geq a_2 \geq \ldots\ldots \geq a_n \geq a_{n+1} \geq \ldots\ldots$ i.e. $\{a_n\}_{n=1}^{\infty}$ is non-increasing and

(*ii*) $\lim_{n \to \infty} a_n = 0$, then prove that the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ is convergent.

(*b*) If $0 < x < 1$, then prove that $\sum_{n=1}^{\infty} x^n$ converges to $\frac{1}{(1-x)}$ and if $x \geq 1$ then prove that $\sum_{n=1}^{\infty} x^n$ diverges.

(*c*) Show that $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converges.