This question paper contains **4** printed pages]

W—92—2018

FACULTY OF ARTS AND SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(CBCS/CGPA)

MATHEMATICS

Paper VII

(Group Theory)

(MCQ & Theory)

(Saturday, 20-10-2018) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) First 30 minutes for Q. No. **1** and remaining time for other questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use black ball pen to darken the circle on OMR sheet for Q. No. **1**.

(*iv*) Negative marking system is applicable for Q. No. **1** (MCQ).

MCQ

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) The mapping τ of S into T is said to one-to-one mapping iff :

(*a*) Whenever $S_1 \neq S_2$, then $S_1\tau \neq S_2\tau$

(*b*) Whenever $S_1 = S_2$, then $S_1\tau = S_2\tau$

(*c*) Whenever $S_1 \neq S_2$, then $S_1\tau = S_2\tau$

(*d*) Whenever $S_1 = S_2$, then $S_1\tau \neq S_2\tau$

P.T.O.

(*ii*) The integers a and b are relatively prime if :

(*a*) $(a, b) = 0$ (*b*) $(a, b) > 1$

(*c*) $(a, b) < 1$ (*d*) $(a, b) = 1$

(*iii*) c is the greatest common divisor of a and b if :

(*a*) b is a divisor of a and c

(*b*) c is a divisor of a and b

(*c*) a is a divisor of b and c

(*d*) c is a not a divisor of a and b

(*iv*) G is a group then which of the following is *true* ?

(*a*) $a, b \in G \Rightarrow a \cdot b \in G$

(*b*) $a, b, c \in G \Rightarrow a \cdot (b \cdot c) = (a \cdot b) \cdot c$

(*c*) There exists an element $e \in G$ such that $a \cdot e = e \cdot a = a$ for all G

(*d*) All of the above

(*v*) Let G be a group, H a subgroup of G. For $a, b \in G$ we say a is congruent to b mod H, then :

(*a*) $a^{-1}b^{-1} \in H$ (*b*) $a^{-1}b \in H$

(*c*) $ab^{-1} \in H$ (*d*) $b^{-1}a^{-1} \in H$

(*vi*) If G is a finite group and H is a subgroup of G, then O(H) is a divisor of O(G). This theorem is known as :

(*a*) Lagrange's theorem (*b*) Euler theorem

(*c*) Fermat theorem (*d*) Cauchy's theorem

(*vii*) Let G be the group S_3 and let H be the subgroup $\{e, \phi\}$, then the number of left cosets of H in G is :

(*a*) 2 (*b*) 3

(*c*) 0 (*d*) 4

(*viii*) If ϕ is a homomorphism of G into $\bar{G}$, the kernel of ϕ, K_ϕ is defined by :

(*a*) $K_\phi = \{x \in G \mid \phi(x) = e,\ e = \text{identity element of } \bar{G}\}$

(*b*) $K_\phi = \{x \in G \mid \phi(x) = \bar{e},\ \bar{e} = \text{identity element of } \bar{G}\}$

(*c*) $K_\phi = \{x \in G \mid \phi(x) = \bar{e},\ \bar{e} = \text{identity element of } G\}$

(*d*) None of the above

(*ix*) If ϕ is a homomorphism of G onto $\bar{G}$ with kernel K, then :

(*a*) $G/K = G$ (*b*) $G/K \approx G$

(*c*) $G/K \approx \bar{G}$ (*d*) $G/K = \bar{G}$

(*x*) The product of two odd permutations is :

(*a*) An even permutation (*b*) An odd permutation

(*c*) Not a permutation (*d*) None of these

Theory

2. Attempt any *two* of the following : 5 each

(*a*) Prove that the relation congruence modulo *n* defines an equivalence relation on the set of integers.

(*b*) If G is a group, then prove that :

(*i*) The identity element of G is unique.

(*ii*) Every $a \in G$ has a unique inverse in G.

(*c*) Let $S = \{x_1, x_2, x_3\}$ and T = S. Let $\sigma : S \to S$ be defined by :

$$x_1\sigma = x_2$$
$$x_2\sigma = x_3$$
$$x_3\sigma = x_1$$

and $\tau : S \to S$ by

$$x_1\tau = x_1$$
$$x_2\tau = x_3$$
$$x_3\tau = x_2$$

Find $\sigma \circ \tau$ and $\tau \circ \sigma$.

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) For all $a \in$ G, prove that :

$$H_a = \{x \in G \mid a \equiv x \bmod H\}.$$

(*b*) Prove that N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \in G$.

(*c*) If G is a group and H is a subgroup of index 2 in G, then prove that H is a normal subgroup of G.

4. Attempt any *two* of the following : 5 each

(*a*) If ϕ is a homomorphism of G into $\bar{G}$ with kernel K, then prove that K is a normal subgroup of G.

(*b*) If G is a group then prove that A(G), the set of automorphisms of G, is also a group.

(*c*) Find the orbits and cycles of the following permutation :

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}.$$