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B—91—2019

FACULTY OF ARTS AND SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) EXAMINATION

MARCH/APRIL, 2019

(CBCS/CGPA Pattern)

MATHEMATICS

Paper VI

(Real Analysis—I)

(MCQ & Theory)

(Thursday, 28-3-2019) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes for Q. No. **1** and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball pen to darken the circle on OMR sheet for Q. No. **1**.

(*v*) Negative marking system is applicable for Q. No. **1**.

MCQ

1. Choose the most *correct* alternative for each of the following : 1 each

(*i*) If set B is countable subset of the uncountable set A, then set A – B is :

(*a*) Countable (*b*) Uncountable

(*c*) Finite (*d*) All of these

P.T.O.

(*ii*) If I be the set of positive integers, then which of the following is its lower bound ?

(*a*) –7 (*b*) 0

(*c*) –1 (*d*) All of these

(*iii*) If $f : A \to B$, then f is called one to one if $f(a_1) = f(a_2)$ implies :

(*a*) A = B (*b*) $a_1 = a_2$

(*c*) $a_1 \neq a_2$ (*d*) A ≠ B

(*iv*) If $f(x) = 1 + \sin x \quad (-\infty < x < \infty)$

$g(x) = x^2 \quad (0 \leq x < \infty)$

then $g \circ f(x) =$ ....

(*a*) $(1 + \sin x)^2, (-\infty < x < \infty)$

(*b*) $(1 + \sin x)^2 + x^2, (0 < x < \infty)$

(*c*) $g(x) + f(x)$

(*d*) All of the above

(*v*) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ has the limit L, then we say, the sequence $\{S_n\}_{n=1}^{\infty}$ is :

(*a*) divergent at L (*b*) convergent to L

(*c*) divergent everywhere (*d*) none of these

(*vi*) All subsequence of a convergent sequence of real numbers converges to the :

(*a*) different point (*b*) 0

(*c*) same limit (*d*) none of these

(*vii*) The sequence $\{n\}_{n=1}^{\infty}$ is :

(*a*) non-increasing sequence (*b*) non-decreasing sequence

(*c*) monotone sequence (*d*) both (*b*) and (*c*) (*viii*) If the series $a_1 + a_2 + ...$ converges to s, then the series $a_2 + a_3 + a_4 + ...$ converges to :

(*a*) s (*b*) $s - a_1$

(*c*) 0 (*d*) All of these

(*ix*) If $\sum_{n=1}^{\infty} a_n$ is a series of non-negative numbers with $S_n = a_1 + a_2 + ... + a_n$, then $\sum_{n=1}^{\infty} a_n$ converges if :

(*a*) Sequence $\{S_n\}_{n=1}^{\infty}$ is not bounded

(*b*) Sequence $\{S_n\}_{n=1}^{\infty}$ is convergent

(*c*) Sequence $\{S_n\}_{n=1}^{\infty}$ is divergent

(*d*) Sequence $\{S_n\}_{n=1}^{\infty}$ is bounded

(*x*) If $x \geq 1$, then the series $\sum_{n=0}^{\infty} x^n$:

(*a*) convergent (*b*) convergent to $1/(1 - x)$

(*c*) both (*a*) and (*b*) (*d*) divergent

Theory

2. Attempt any *two* of the following : 5 each

(*a*) If $f : A \to B$ and if $X \subset B$, $Y \subset B$ then prove that :

$$f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y).$$

(*b*) Prove that the set :

$$[0, 1] = \{x \mid 0 \leq x \leq 1\}$$

is uncountable.

(*c*) Prove that any infinite set contains a countable subset.

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) If $\{S_n\}_{n=1}^{\infty}$ is a sequence of non-negative numbers and if $\lim_{n \to \infty} S_n = L$, then prove that $L \geq 0$.

(*b*) Prove that :

$$\lim_{n \to \infty} \frac{3n^2 - 6n}{5n^2 + 4} = \frac{3}{5}.$$

(*c*) If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers, then prove that :

$$\lim_{n \to \infty} \sup S_n = \lim_{n \to \infty} S_n.$$

4. Attempt any *two* of the following : 5 each

(*a*) Prove that the series :

$$\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)$$

is divergent.

(*b*) If $\sum_{n=1}^{\infty} a_n$ converges absolutely to A, then prove that any rearrangement $\sum_{n=1}^{\infty} b_n$ of $\sum_{n=1}^{\infty} a_n$ also converges absolutely to A.

(*c*) Show that the series :

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{(2n-1)}$$

diverges.