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X—56—2019

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. (Second Year) (Third Semester) (Regular) EXAMINATION

OCTOBER/NOVEMBER, 2019

MATHEMATICS

Paper VIII

(Ordinary Differential Equations)

(Monday, 18-11-2019) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

(*iii*) Attempt (A) *or* (B) (*a*), (*b*) in Question No. **1** and **2**.

1. Attempt (A) *or* (B) of the following :

(A) If r is a root of multiplicity m of a polynomial p, deg $p \geq 1$, then prove that : 15

$$p(r) = p'(r) = ... = p^{(m-1)}(r) = 0,$$

and

$$p^{(m)}(r) \neq 0.$$

If r is such that $r^3 = 1$, and $r \neq 1$, then prove that $1 + r + r^2 = 0$.

Or

(B) (*a*) Consider the equation 8

$$y' + ay = 0,$$

where a is a complex constant. If c is any complex number, then prove that the function ϕ defined by

$$\phi(x) = ce^{-ax}$$

is a solution of this equation, and every solution has this form.

P.T.O.

(*b*) Consider the equation 7

$$y' + 5y = 2.$$

Show that the function ϕ given by

$$\phi(x) = \frac{2}{5} + ce^{-5x}$$

is a solution, where c is any constant. Assuming every solution has this form, find the solution satisfying $\phi(1) = 2$. Find that solution satisfying $\phi(1) = 3\phi(0)$.

2. Attempt (A) *or* (B) of the following :

(A) Let a_1, a_2 be constants, and consider the equation 15

$$L(y) = y'' + a_1 y' + a_2 y = 0.$$

If r_1, r_2 are distinct roots of the characteristic polynomial p, where

$$p(r) = r^2 + a_1 r + a_2,$$

then prove that ϕ_1, ϕ_2 defined by

$$\phi_1(x) = e^{r_1 x},\ \phi_2(x) = e^{r_2 x},$$

are solutions of $L(y) = 0$. If r_1 is a repeated root of p, then the functions ϕ_1, ϕ_2 defined by

$$\phi_1(x) = e^{r_1 x},\ \phi_2(x) = xe^{r_1 x}$$

are solutions of $L(y) = 0$.

Or

(B) (*a*) Prove that, two solutions ϕ_1, ϕ_2 of $L(y) = y'' + a_1 y' + a_2 y = 0$ are linearly independent on an interval I, if, and only if,

$$W(\phi_1, \phi_2)(x) \neq 0$$

for all x in I. 8

(*b*) Find all the solutions of $y'' - y' - 2y = e^{-x}$. 7

3. Attempt any *two* of the four of the following : 5 each

(*a*) Prove that there exist n linearly independent solutions of

$$L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \ldots\ldots + a_n(x)y = 0$$

on I.

(*b*) Let $\phi_1, \phi_2, \ldots\ldots, \phi_n$ be n linearly independent solutions of

$$L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \ldots\ldots + a_n(x)y = 0$$

on an interval I. If ϕ is any solution of $L(y) = 0$ on I, then prove that ϕ can be represented in the form

$$\phi = c_1\phi_1 + c_2\phi_2 + \ldots\ldots + c_n\phi_n.$$

(*c*) Consider the equation

$$y'' + \frac{1}{x}y' - \frac{1}{x^2}y = 0$$

for $x > 0$. Show that there is a solution of the form x^r, where r is a constant.

(*d*) Consider the equation $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$. If ϕ_1 and ϕ_2 are linearly independent solutions of $L(y) = 0$, then prove that :

$$a_1 = \frac{\begin{vmatrix} \phi_1 & \phi_2 \\ \phi_1'' & \phi_2'' \end{vmatrix}}{W(\phi_1, \phi_2)} \text{ and } a_2 = \frac{\begin{vmatrix} \phi_1' & \phi_2' \\ \phi_1'' & \phi_2'' \end{vmatrix}}{W(\phi_1, \phi_2)}.$$