This question paper contains **4** printed pages]

Y—108—2019

FACULTIES OF ARTS AND SCIENCE

B.A./B.Sc. (Second Year) (Third Semester) (Backlog) EXAMINATION

NOVEMBER/DECEMBER, 2019

(CBCS/CGPA Pattern)

MATHEMATICS

Paper VII

(Group Theory)

(MCQ + Theory)

(Thursday, 19-12-2019) **Time : 2.00 p.m. to 4.00 p.m.**

Time—Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes for Question No. 1 and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball point pen to darken the circle on OMR sheet for question No. 1.

(*v*) Negative marking system is applicable for Q. No. 1.

MCQ 10

1. Choose the *correct* alternative for each of the following :

(1) If S be any non-empty set, define $i : S \to S$ by $s = st$ for any $s \in S$. This mapping i is called .... mapping of S.

(A) Identity (B) Many one

(C) Inverse (D) None of these

P.T.O.

(2) The integer $p > 1$ is a prime number if its only divisors are ....

(A) ±1 (B) ±p

(C) Both (A) and (B) (D) None of these

(3) If G be the set of all real 2 × 2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $ad - bc \neq 0$ is a rational number, then G forms .... under matrix multiplication.

(A) an abelian group (B) a group

(C) Both (A) and (B) (D) None of these

(4) A non-empty subset H of the group G is a subgroup of G, if and only if ....

(A) $a, b \in H \Rightarrow ab \in H$ (B) $a \in H \Rightarrow a^{-1} \in H$

(C) Both (A) and (B) (D) None of these

(5) If p is a prime number and a is any integer, then ....

(A) $a^p \equiv p \bmod a$ (B) $p^a \equiv a \bmod p$

(C) $a^p \equiv a \bmod n$ (D) $a^p \equiv a \bmod p$

(6) A subgroup N of G is said to be a normal subgroup of G if for every $g \in G$ and $n \in N$, ...

(A) $n g n^{-1} \in N$ (B) $N g N^1 \in n$

(C) $g n g \in N$ (D) $g n g^{-1} \in N$

(7) A mapping ϕ from a group G into $\bar{G}$ is said to be a homomorphism if for all $a, b \in G$, ...

(A) $\phi(ab) = \phi(a)\,\phi(b)$ (B) $\phi(a+b) = \phi(a) + \phi(b)$

(C) Both (A) and (B) (D) $\phi(a+b) = \phi(a)\,\phi(b)$

(8) If ϕ be a homomorphism of G onto $\bar{G}$ with kernal K, and let $\bar{N}$ be a normal subgroup of $\bar{G}$,

$N = \{x \in G \mid \phi(x) \in \bar{N}\}$, then ....

(A) $N/G \approx \bar{N}/\bar{G}$ (B) $\bar{G}/\bar{N} \approx N/G$

(C) $\bar{G}/N \approx \bar{G}/\bar{N}$ (D) $G/N \approx \bar{G}/\bar{N}$

(9) Every permutation can be uniquely expressed as a product of .... cycles.

(A) Joint (B) Disjoint

(C) Three (D) None of these

(10) The permutation given by the cycle (1, 2, 3) is of which type ?

(A) An even permutation (B) A 2-cycle

(C) An odd permutation (D) None of these

Theory

2. Attempt any *two* of the following : 10

(*a*) If $\sigma : S \to T$ and $\tau : T \to U$; then prove that $\sigma \circ \tau$ is one-to-one if each of σ and τ is one-to-one.

(*b*) If a is relatively prime to b but $a \mid bc$, then prove that $a \mid c$.

(*c*) Show that if every element of the group G is its own inverse, then G is abelian.

3. Attempt any *two* of the following : 10

(*a*) If G is a finite group and $a \in G$, then prove that $0(a) \mid 0(G)$.

(*b*) Show that the relation $a \equiv b \bmod H$ is an equivalence relation.

(*c*) If H is a subgroup of G and N is a normal subgroup of G, then show that $H \cap N$ is a normal subgroup of H.

P.T.O.

4. Attempt any *two* of the following : 10

(*a*) If G is a group, N a normal subgroup of G; define the mapping ϕ from G to $G|N$ by $\phi(x) = Nx$ for all $x \in G$. Then prove that ϕ is a homomorphism of G into $G|N$.

(*b*) If G is a group, then prove that A(G), the set of automorphism of G, is also a group.

(*c*) Find the orbit and cycles of the permutation :

$$\theta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 8 & 1 & 6 & 4 & 7 & 5 & 9 \end{pmatrix}.$$