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Y—131—2019

FACULTY OF SCIENCE

B.Sc. (Second Year) (Third Semester) (Backlog) EXAMINATION

NOVEMBER/DECEMBER, 2019

MATHEMATICS

Paper VIII

(Ordinary Differential Equations)

(MCQ & Theory)

(Saturday, 21-12-2019) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) First 30 minutes for Q. No. **1** and remaining time for other questions.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use black ball point pen to darken the circle on OMR sheet for Q. No. **1**.

(*iv*) Negative marking system is applicable for Q. No. **1**.

(*v*) *All* questions are compulsory.

MCQ

1. Choose the *correct* alternative for each of the following : 10

(*i*) If P is a polynomial, $\deg p = n \geqq 1$, with leading coefficient $a_0 \neq 0$, then P has :

(*a*) At least one root (*b*) Exactly n root

(*c*) One or two roots (*d*) Infinitely many roots

(*ii*) The determinant of the coefficients of the system of equations :

$3z_1 + z_2 - z_3 = 0$, $2z_1 - z_3 = 1$, $z_2 + 2z_3 = 2$ is :

(*a*) –3 (*b*) 3

(*c*) 2 (*d*) 0

P.T.O.

(*iii*) A boundary condition is a condition on the solution at :

(*a*) Singular and regular points

(*b*) One point

(*c*) Two or more points

(*d*) Pole.

(*iv*) The equation $y' + a(x)y = b(x)$ where $b(x) = 0$ on I, is called :

(*a*) Homogeneous first order differential equation

(*b*) Non-homogeneous first order differential equation

(*c*) Homogeneous second order differential equation

(*d*) Non-homogeneous second order differential equation

(*v*) If r_1, r_2 are distinct roots of the characteristic polynomial P, where $P(r) = r^2 + a_1 r + a_2$, then the functions ϕ_1, ϕ_2 defined by :

(*a*) $\phi_1(x) = e^{r_1 x}$ and $\phi_2(x) = xe^{r_1 x}$

(*b*) $\phi_1(x) = e^{r_1 x}$ and $\phi_2(x) = e^{-r_2 x}$

(*c*) $\phi_1(x) = e^{-r_1 x}$ and $\phi_2(x) = e^{-r_2 x}$

(*d*) $\phi_1(x) = e^{r_1 x}$ and $\phi_2(x) = e^{r_2 x}$

(*vi*) If ϕ_1, ϕ_2 are any two solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$, c_1, c_2 are any two constants, then the function $\phi = c_1\phi_1 + c_2\phi_2$ is :

(*a*) Not solution of L(*y*) = 0

(*b*) A solution of L(*y*) = 0

(*c*) Characteristic polynomial of L(*y*) = 0

(*d*) None of the above

(*vii*) If ϕ is a solution of the equation $y'' + \frac{2}{3}y' = 0$, then ϕ = ...

(*a*) $\phi = c_1 + c_2 e^{\frac{2}{3}x}$ (*b*) $\phi = c_1 + c_2 e^{\frac{-2}{3}x}$

(*c*) $\phi = c_1 - c_2 e^{\frac{-2}{3}x}$ (*d*) $\phi = c_1 - c_2 e^{\frac{2}{3}x}$

(*viii*) In the equation : $a_0(x)y^{(n)} + a_1(x)y^{(n-1)} + ... + a_n(x)y = b(x)$. The value of x are called singular point if :

(*a*) $a_0(x) = 1$ (*b*) $a_0(x) = -1$

(*c*) $a_0(x) \neq 0$ (*d*) $a_0(x) = 0$

(*ix*) Let $b_1, b_2, ..., b_n$ be non-negative constants such that for all x in I

$$|a_j(x)| \leqq b_j, \ (j = 1, 2, ..., n),$$

and define k by

$$k = 1 + b_1 + b_2 + ... + b_n.$$

If x_0 is a point in I, and ϕ is a solution of :

$$L(y) = y^{(n)} + a_1(x)y^{(n-1)} + ... + a_n(x)y = 0 \text{ on I, then :}$$

(*a*) $||\phi(x)||e^{k|x-x_0|} \leq ||\phi(x)|| \leq ||\phi(x_0)||e^{-k|x-x_0|}$

(*b*) $||\phi(x)||e^{-k|x-x_0|} \leq ||\phi(x)|| \leq ||\phi(x_0)||e^{-k|x-x_0|}$

(*c*) $||\phi(x_0)||e^{-k|x-x_0|} \leq ||\phi(x)|| \leq ||\phi(x_0)||e^{k|x-x_0|}$

(*d*) None of the above

(*x*) The solution ϕ of $xy' + y = 0$ s.t. $y(1) = 1$ is given by :

(*a*) $\phi(x) = x$ (*b*) $\phi(x) = \frac{1}{x}$

(*c*) $\phi(x) = \frac{1}{x^2}$ (*d*) None of these

Theory

2. Attempt any *two* of the following : 5 each

(*a*) If r is a root of multiplicity m of a polynomial p, deg $p \geqq 1$, then prove that : $p(r) = p'(r) = ... = p^{(m-1)}(r) = 0$, and $p^{(m)}(r) \neq 0$.

P.T.O.

(*b*) Consider the equation : $y' + ay = 0$, where a is a complex constant. If c is any complex number, then show that the function ϕ defined by

$$\phi(x) = ce^{-ax}$$

is a solution of this equation and moreover every solution has this form.

(*c*) Consider the equation $y' + 5y = 2$.

(*a*) Show that the function ϕ given by

$$\phi(x) = \frac{2}{5} + ce^{-5x}$$

is a solution, where c is any constant.

(*b*) Assuming every solution has this form, find that solution satisfying $\phi(1) = 2$.

3. Attempt any *two* of the following : 5 each

(*a*) For any real x_0, and constants α, β, prove that there exists a solution ϕ of the initial value problem :

$$L(y) = y'' + a_1 y' + a_2 y = 0,$$

$$y(x_0) = \alpha, \ y'(x_0) = \beta,$$

on $-\infty < x < \infty$.

(*b*) If ϕ_1, ϕ_2 are two solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing a point x_0, then prove that :

$$w(\phi_1, \phi_2)(x) = e^{-a_1(x - x_0)} w(\phi_1, \phi_2)(x_0)$$

(*c*) Consider the equation $y'' + y' - 6y = 0$.

(*a*) Compute the solution ϕ satisfying

$$\phi(0) = 1, \ \phi'(0) = 0$$

(*b*) Compute the solution ψ satisfying

$$\psi(0) = 0, \ \psi'(0) = 1.$$

4. Attempt any *two* of the following : 5 each

(*a*) Prove that there exists n linearly independent solution of :

$$L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \ldots + a_n(x) = 0$$

on an interval I.

(*b*) Let $\phi_1, \ldots, \phi_n$ be n linearly independent solution of :

$$L(y) = y^{(n)} + a_1(x)y^{(n-1)} + \ldots + a_n(x) = 0$$

on an interval I. If ϕ is any solution of $L(y) = 0$ on I, then prove that ϕ can be represented in the form :

$$\phi = c_1\phi_1 + c_2\phi_2 + \ldots + c_n\phi_n$$

where $c_1, c_2, \ldots, c_n$ are constants.

(*c*) Find two linearly independent solutions of the equation

$$(3x-1)^2 y'' + (9x-3)y' - 9y = 0$$

for $x > \frac{1}{3}$.