This question paper contains **4** printed pages]

Y—91—2019

FACULTY OF SCIENCE/ARTS

B.Sc./B.A. (Second Year) (Third Semester) (Backlog) EXAMINATION

OCTOBER/NOVEMBER, 2019

MATHEMATICS

Paper VI

(Real Analysis-I)

(MCQ + Theory)

(Monday, 18-11-2019) **Time : 2.00 p.m. to 4.00 p.m.**

Time— Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes for Q. No. **1** and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ink ball pen to darken the the circles on OMR sheet for Q. No. 1.

(*v*) Negative marking system is applicable for Q. No. **1** (MCQ).

(MCQ)

1. Choose the most correct alternative for each of the following : 10

(*i*) Which of the following need *not* be true ?

(*a*) $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$

(*b*) $f(X \cap Y) = f(X) \cap f(Y)$

(*c*) $f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$

(*d*) $f(X \cup Y) = f(X) \cup f(Y)$

(*ii*) Let A = [0, 1]. If $f : A \to \mathbf{R}$ is defined by $f(x) = x$ and $g : A \to \mathbf{R}$ is defined by $g(x) = x^2$, then $\min(f, g)(x) = ?$

(*a*) 0 (*b*) 1

(*c*) x (*d*) x^2

P.T.O.

(*iii*) Let P be the set of prime integers. Which of the following is true ?

(*a*) $7 \in P$

(*b*) $9 \in P$

(*c*) $11 \notin P$

(*d*) $7547193.65317 \in P$

(*iv*) Let $\{S_n\}_{n=1}^{\infty}$ be a sequence of real numbers. Then :

(*a*) $\lim_{n\to\infty} \inf S_n \geq \lim_{n\to\infty} \sup S_n$

(*b*) $\lim_{n\to\infty} \inf S_n \leq \lim_{n\to\infty} \sup S_n$

(*c*) $\lim_{n\to\infty} \inf S_n = \lim_{n\to\infty} \sup S_n$

(*d*) $\lim_{n\to\infty} S_n > 0$

(*v*) Consider the following *two* statements :

(*i*) Every convergent sequence of real numbers is bounded.

(*ii*) Every bounded sequence of real numbers is convergent.

Then :

(*a*) Both (*i*) and (*ii*) are the true

(*b*) (*i*) is true and (*ii*) is false

(*c*) (*i*) is false and (*ii*) is true

(*d*) both (*i*) and (*ii*) are false

(*vi*) The limit of sequence $\left\{n - \frac{1}{n}\right\}_{n=1}^{\infty}$ ....

(*a*) is 0

(*b*) is finite

(*c*) doesn't exist

(*d*) –1

(*vii*) For the series $\sum_{n=1}^{\infty} a_n$, if $\lim_{n\to\infty} a_n = 0$, then :

(*a*) $\sum_{n=1}^{\infty} a_n$ is convergent

(*b*) $\sum_{n=1}^{\infty} a_n$ is divergent

(*c*) $\sum_{n=1}^{\infty} a_n$ may or may not convergent

(*d*) none of the above

(*viii*) The series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\ldots\ldots$ is :

(*a*) divergent (*b*) absolutely convergent

(*c*) conditionally convergent (*d*) all of these

(*ix*) Which of the following is *true* ?

(*a*) $\sum_{n=1}^{\infty} n$ is convergent (*b*) $\sum_{n=1}^{\infty}\frac{1}{n}$ is convergent

(*c*) $\sum_{n=1}^{\infty}\frac{n}{n+1}$ is convergent (*d*) $\sum_{n=1}^{\infty}\frac{1}{n}$ is divergent

(*x*) Let $\sum_{n=1}^{\infty} a_n$ be a series of non-zero real numbers and let :

$$a=\liminf_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|,$$

$$\mathrm{A}=\limsup_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|.$$

Then which of the following is *true* ?

(*a*) If A < 1, then $\sum_{n=1}^{\infty}|a_n|<\infty$

(*b*) If a > 1, then $\sum_{n=1}^{\infty} a_n$ diverges

(*c*) If $a \le 1 \le \mathrm{A}$, then the test fails

(*d*) All of the above

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) If f : A → B and X ⊂ A, Y ⊂ A then prove that :

$f(\mathrm{X} \cup \mathrm{Y}) = f(\mathrm{X}) \cup f(\mathrm{Y})$.

P.T.O.

(*b*) Prove that the set R of all real numbers is uncountable.

(*c*) Prove that the set of all polynomial functions with integers coefficients is countable.

3. Attempt any *two* of the following : 5 each

(*a*) If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent, then prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.

(*b*) If $\{s_n\}_{n=1}^{\infty}$ is bounded sequence of real numbers and $\{t_n\}_{n=1}^{\infty}$ convergent to 0, prove that $\{s_n t_n\}_{n=1}^{\infty}$ converges to 0.

(*c*) If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers, then prove that :

$$\lim_{n\to\infty} \inf s_n \le \lim_{n\to\infty} \sup s_n .$$

4. Attempt any *two* of the following : 5 each

(*a*) If $\sum_{n=1}^{\infty} a_n$ is a series of non-negative numbers with $s_n = a_1 + a_2 + \ldots\ldots\ldots\ldots + a_n$ $(n \in \mathrm{I})$. Then prove that $\sum_{n=1}^{\infty} a_n$ converges if, the sequence $\{s_n\}_{n=1}^{\infty}$ is bounded.

(*b*) If $\sum_{n=1}^{\infty} a_n$ converges absolutely, then prove that $\sum_{n=1}^{\infty} a_n$ converges.

(*c*) Prove that if $\sum_{n=1}^{\infty} |a_n| < \infty$, then $\left|\sum_{n=1}^{\infty} a_n\right| \le \sum_{n=1}^{\infty} |a_n|$.