This question paper contains **8+3** printed pages]

BF—62—2016

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

(Old Course)

MATHEMATICS

Paper IX

(Ordinary Differential Equations)

(MCQ)

(Tuesday, 18-10-2016) **Time : 2.00 p.m. to 3.00 p.m.**

Time—1 *Hour* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) *All* questions carry equal marks.

(*iii*) Wrong answers carry negative marks.

1. If p is a polynomial deg $p = n \geqq 1$ with leading coefficient $a_0 \neq 0$, then p has exactly n roots. If $r_1, r_2, \ldots\ldots r_n$ are roots of p, then :

(A) $p(z) = a_0(z - r_1)(z - r_2) \ldots\ldots (z - r_n)$

(B) $p(z) = a_0 / (z - r_1)(z - r_2) \ldots\ldots (z - r_n)$

(C) $p(z) = \frac{a_0}{2}(z - r_1)(z - r_2) \ldots\ldots (z - r_n)$

(D) $p(z) = \frac{a_0}{n}(z - r_1)(z - r_2) \ldots\ldots (z - r_n)$

2. If $p(z)$ is a polynomial of degree 3 with real coefficients, then p has :

(A) At least two imaginary roots (B) At least one real root

(C) At least one imaginary root (D) At least two real roots

P.T.O.

3. Consider a system of n equations :

$$\begin{aligned} a_{11} z_1 + a_{12} z_2 + \dots\dots\dots + a_{1n} z_n &= c_1 \\ a_{21} z_1 + a_{22} z_2 + \dots\dots\dots + a_{2n} z_n &= c_2 \\ \vdots \qquad \vdots \qquad\qquad \vdots \quad &\quad \vdots \\ a n_1 z_1 + a_{n2} z_2 + \dots\dots\dots + a_{nn} z_n &= c_n \\ \text{if } c_1 = c_2 = c_3 = \dots\dots\dots = c_n &= 0 \end{aligned}$$

then :

(A) It is non-homogeneous system of n linear equations

(B) It is homogeneous system of n linear equations

(C) It is a polynomial has degree $n - 1$

(D) It is a polynomial of degree $n + 1$

4. The characteristic polynomial of equation :

$$y'' + y' - 6y = 0$$

is :

(A) $r - 6$ (B) $r^2 + r + 6$

(C) $r^2 + r - 6$ (D) $r^2 - r - 6$

5. Consider the system of equations :

$$iz_1 + z_2 = 1 + i$$
$$2z_1 + (2 - i) z_2 = 1$$

The value of z_1 and z_2 are :

(A) $i/2$ and $-i/2$ (B) $2i$ and $3i$

(C) i and $i/2$ (D) $-i$ and i

6. If p is a polynomial deg $p \geq 1$ and r is complex number such that :

$$p(r) = p'(r) = \dots = p^{(m-1)}(r) = 0,\ p^{(m)}(r) \neq 0$$

then :

(A) r is a root of p with multiplicity m

(B) r is a root of p with multiplicity $m - 1$

(C) r is a root of p with multiplicity $m + 1$

(D) r is a root of p with multiplicity $m + 2$

7. A boundary condition is a condition on the solution at :

(A) One point

(B) Two or less than two points

(C) Two or more points

(D) Infinite points

8. Let ϕ be the solution of $y' + iy = x$ such that $\phi(0) = 2$, then what is $\phi(\pi)$?

(A) πi (B) $-\pi i$

(C) $\pi/2$ (D) $-\pi/2$

9. Which of the following is the solution of differential equation $y'' + y = 0$?

(A) $\phi(x) = a \sin x + b \cos x$ (B) $\phi(x) = \cos x$

(C) $\phi(x) = \sin x$ (D) All of these

10. If $\phi(x) = e^x$ is a solution of :

(A) $y'' - y = 0$ (B) $y'' + y = 0$

(C) $y'' + y' = 0$ (D) $y' + y = 0$

11. $y' + 2xy = x$ has a solution ϕ, then required solution $\phi(x)$ is ....

(A) $\phi(x) = \frac{1}{2} + ce^{-x^2}$ (B) $\phi(x) = -\frac{1}{2} + ce^{x^2}$

(C) $\phi(x) = -\frac{1}{2} - ce^{-x^2}$ (D) $\phi(x) = \frac{1}{2} + ce^{x^2}$

12. The equation $y' + \alpha(x)y = \beta(x)y^k$ (where k is constant) is known as :

(A) Exact equation (B) Ricatti equation

(C) Bernoulli's equation (D) Fermat equation

13. $\phi(x) = \dfrac{E\omega L}{R^2 + \omega^2 L^2} e^{(-R/L)x} + \dfrac{E}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega x - \alpha)$

is solution of differential equation :

(A) $Ly' + Ry = E \sin \omega x$

(B) $Ly' + Ry = E$

(C) $Ly'' + Ry = E/2$

(D) $Ly' - Ry = E$

14. Consider the equation $y'' = 3x + 1$ has a solution ϕ. What is required solution $\phi(x)$ on $0 \leq x \leq 1$?

(A) $\phi(x) = \dfrac{5}{2}(1 + 4e^{k(1-x)})$

(B) $\phi(x) = \left(\dfrac{2}{5}\right)(1 + 4e^{k(1-x)})$

(C) $\phi(x) = -\dfrac{2}{5}(1 + 4e^{k(1-x)})$

(D) None of these

15. The solution of the differential equation :

$$y' + (\cos x)y = \sin x \cos x$$

is :

(A) $\phi(x) = (1 + \sin x) + ce^{\sin x}$

(B) $\phi(x) = (\sin x - 1) + ce^{\sin x}$

(C) $\phi(x) = (\sin x - 1) + ce^{-\sin x}$

(D) $\phi(x) = (1 - \sin x) + ce^{\sin x}$

16. $L(y) = y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y = b(x)$

is a linear differential equation of order n with constant coefficient if $b(x) = 0$, then it is known as :

(A) Homogeneous equation

(B) Non-homogeneous equation

(C) Second order homogeneous equation

(D) Second order non-homogeneous equation

17. Let a_1, a_2 are constants and consider the equation :

$$L(y) = y'' + a_1 y' + a_2 y = 0.$$

If r_1, r_2 are distinct roots of the characteristic polynomial p. Then required solutions ϕ_1, ϕ_2 are :

(A) $\phi_1(x) = e^{r_1 x}, \phi_2(x) = e^{r_2 x}$
(B) $\phi_1(x) = e^{-r_1 x}, \phi_2(x) = e^{r_2 x}$
(C) $\phi_1(x) = e^{r_1 x}, \phi_2(x) = xe^{r_1 x}$
(D) $\phi_1(x) = e^{-r_1 x}, \phi_2(x) = xe^{r_1 x}$

18. The solution of the differential equation $y'' - 4y' + 5y = 0$ is :

(A) $\phi(x) = e^{-2x}(c_1 \cos x - c_2 \sin x)$
(B) $\phi(x) = e^{2x}(c_1 \cos x + c_2 \sin x)$
(C) $\phi(x) = e^{-2x}(c_1 \cos x + c_2 \sin x)$
(D) $\phi(x) = e^{x}(c_1 \cos x + c_2 \sin x)$

19. All solution ϕ of $y'' + y = 0$ satisfying :

$$\phi(0) = 1, \phi(\pi / 2) = 2$$

is :

(A) $\phi(x) = e^{-x} - e^{x}$
(B) $\phi(x) = e^{x} + e^{-x}$
(C) $\phi(x) = -\frac{1}{2}(e^{x} + e^{-x})$
(D) $\phi(x) = \frac{1}{2}(e^{x} - e^{-x})$

20. What is the characteristic polynomial of differential equation :

$$y'' + \omega^2 y = 0$$

(A) $p(r) = r^2 - w^2$
(B) $p(r) = r^2 + w^2$
(C) $p(r) = \frac{1}{2}(r^2 + w^2)$
(D) $p(r) = w^2 - r^2$

21. The solution of the differential equation :

$$y'' + y' - 2y = 0$$

is :

(A) $\phi(x) = c_1 e^x + c_2 e^{2x}$

(B) $\phi(x) = c_1 e^x + c_2 e^{-2x}$

(C) $\phi(x) = (c_1 + c_2) e^x$

(D) None of these

22. For any real x_0 and constants α, β there exists a solution ϕ of the initial value problem :

$$L(y) = \sigma,\ y(x_0) = \alpha,\ y'(x_0) = \beta \quad \text{on} \quad -\infty < x < \infty$$

It is known as :

(A) Fundamental theorem of algebra

(B) Existence theorem

(C) Uniqueness theorem

(D) All of the above

23. The solution of $y'' + 16y = 0$ is :

(A) $\phi(x) = c_1 \cos 4x + c_2 \sin 4x$

(B) $\phi(x) = c_1 e^{4x} + c_2 e^{-4x}$

(C) $\phi(x) = (c_1 + c_2 x) e^{-4x}$

(D) $\phi(x) = (c_1 + \frac{1}{x} c_2) e^{4x}$

24. The equation $y' + a(x) y = 0$ has a solution :

(A) $\phi(x) = \int_0^{x_0} a(t) dt$

(B) $\phi(x) = \exp\left[\int_{x_0}^{x} a(t) dt\right]$

(C) $\phi(x) = \exp\left[-\int_{x_0}^{x} a(t) dt\right]$

(D) $\phi(x) = \exp\left[\int_0^{x} a(t) dt\right]$

25. The solution of the equation :

$$y'' + \frac{1}{x} y' - \frac{y}{x^2} = 0$$

(in the form x^r) is :

(A) $r_1 = 1, r_2 = -1$

(B) $r_1 = 2, r_2 = 3$

(C) $r_1 = 1, r_2 = 2$

(D) $r_1 = -1, r_2 = \frac{1}{2}$

26. The function ϕ_1, ϕ_2 are said to be linearly dependent on any interval I if :

(A) $c_1\phi_1 + c_2\phi_2 = 0$ if $c_1, c_2 = 0$

(B) $c_1 \phi_1 + c_2 \phi_2 = 0$, c_1, c_2 not both zero

(C) Both (A) and (B)

(D) None of the above

27. Two functions ϕ_1, ϕ_2 are solution of L(y) = 0 on an interval I and x_0 be any point in I ϕ_1, ϕ_2 are linearly independent if and only if :

(A) $W(\phi_1, \phi_2)(x_0) = 1$

(B) $W(\phi_1, \phi_2)(x_0) = 0$

(C) $W(\phi_1, \phi_2)(x_0) \neq 0$

(D) $W(\phi_1, \phi_2)(x_0) = \infty$

28. Two functions ϕ_1, ϕ_2 are defined by :

$$\phi_1(x) = x \text{ and } \phi_2(x) = e^{rx},$$

r is complex constant. Then functions are :

(A) Linearly dependent

(B) Linearly independent

(C) Both (A) & (B)

(D) None of the above

29. Let ϕ_1, ϕ_2 be any two linearly independent solutions of $L(y) = 0$ on an interval I. Every solution ϕ of $L(y) = 0$ can be uniquely expressed as :

(A) $\phi = c_1 \phi_1 + c_2 \phi_2$

(B) $\phi = (c_1 + c_2)\ \phi_1 (x)$

(C) $\phi = \frac{c_1}{\phi_1} + \frac{c_2}{\phi_2}$

(D) $\phi = c_1 \phi_1 / c_2 \phi_2$

30. Let α, β be any *two* constants and let x_0 be any real number. On any interval I containing x_0 there exists at most one solution ϕ of initial value problem :

$$L(y) = \sigma,\ y(x_0) = \alpha,\ y'(x_0) = \beta.$$

It is known as :

(A) Existence theorem

(B) Uniqueness theorem

(C) Fundamental theorem of algebra

(D) Fermat theorem

31. Let b be continuous on an interval I. Every solution ψ of :

$$L(y) = y'' + a_1 y' + a_2 y = b(x)$$

on I can be represented as :

$$\psi = \psi_p + c_1 \phi_1 + c_2 \phi_2$$

it is called :

(A) Basic solution

(B) Particular solution

(C) Complementary function

(D) Complete solution

32. The equation :

$$M(x, y)\ dx + N(x, y)\ dy = 0$$

is exact if and only if :

(A) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(B) $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$

(C) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(D) $\frac{\partial M}{\partial x} = -\frac{\partial N}{\partial y}$

33. If :

$$f(x, y) = \frac{g(x)}{h(y)}$$

then the equation $y' = f(x, y)$ is called :

(A) Only x variable separated

(B) Only y variable separated

(C) Non-variable separated

(D) Variables separated

34. The solution of the equation $y' = 3y^{2/3}$ which of the among :

(A) $\phi(x) = (x + c)^4$

(B) $\phi(x) = (x + c)^2$

(C) $\phi(x) = (x + c)^3$

(D) $\phi(x) = \frac{1}{(x + c)^3}$

35. Which of the following is a solution of :

$$y' = \frac{3x^2 - 2xy}{x^2 - 2y}$$

(A) $x^3 + x^2y + y^2 = c$

(B) $x^3 - x^2y + y^2 = c$

(C) $x^3 - y^3 = c$

(D) $x^3 + y^3 = c$

36. If r_1 and r_2 are repeated roots of the characteristic polynomial :

$$p(r) = r^2 + a_1 r + a_2$$

has a solution :

(A) $\phi_1(x) = e^{r_1 x}, \ \phi_2(x) = xe^{r_1 x}$

(B) $\phi_1(x) = e^{r_1 x}, \ \phi_2(x) = e^{r_2 x}$

(C) $\phi_1(x) = e^{-r_1 x}, \ \phi_2(x) = e^{r_2 x}$

(D) $\phi_1(x) = e^{(r_1 + r_2)x}, \ \phi_2(x) = e^{r_2 x}$

37. If ϕ is any solution of :

$$y'' + a_1 y' + a_2 y = 0$$

on an interval I containing a point x_0. Then for all x in I which of the following is *correct* ?

(A) $\|\phi(x_0)\| e^{k|x - x_0|} \leqq \|\phi(x)\| \leqq \|\phi(x_0)\| e^{-k|x - x_0|}$

(B) $\|\phi(x_0)\| e^{-k|x - x_0|} \leqq \|\phi(x)\| \leqq \|\phi(x_0)\| e^{k|x - x_0|}$

(C) $\|\phi(x)\| \leqq \|\phi(x_0)\| \leqq \|\phi(x_0)\| e^{-k(x - x_0)}$

(D) All of the above

38. The ordinary differential of first order is represented by :

(A) $y' = f(x, y)$

(B) $y' = (x, x^2)$

(C) $y' = f(x, y, y')$

(D) $y^{(n)} = f(x, y, y', \ldots\ldots y^{(n-1)}$

39. The equation :

$$L(y) = y'' + a_1(x)y' + a_2 y = 0$$

if ϕ is any solution of $L(y) = 0$ if and only if p satisfies the first order non-linear equation :

$$y' = -y^2 - a_1(x)y - a_2(x)$$

is known as :

(A) Bernoulli's equation

(B) Ricatti equation

(C) Legendre's equation

(D) Fermat equation

40. The solution of the equation $y'' + 10y = 0$ with initial condition :

$$y(0) = \pi,\ y'(0) = \pi^2$$

is :

(A) $\phi(x) = \pi \cos \sqrt{10x} + \dfrac{\pi^2}{\sqrt{10}} \sin \sqrt{10}\, x$

(B) $\phi(x) = -\pi \cos \sqrt{10x} + \dfrac{\pi^2}{\sqrt{10}} \sin \sqrt{10}\, x$

(C) $\phi(x) = \pi \sin \sqrt{10x} + \dfrac{\pi^2}{\sqrt{10}} \cos \sqrt{10}\, x$

(D) $\phi(x) = \pi \sin \sqrt{10x} - \dfrac{\pi}{\sqrt{10}} \cos \sqrt{10}\, x$