This question paper contains **4+2** printed pages]

BF—63—2016

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

(Revised Course)

MATHEMATICS

Paper IX

(Real Analysis—II)

(MCQ + Theory)

(Tuesday, 18-10-2016) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :—(*i*) *All* questions are compulsory.

(*ii*) First **30** minutes are for Q. No. **1** (MCQ) and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball point pen to darken the circle of correct choice in OMR answer-sheet.

(*v*) Negative marking system is applicable for MCQ.

MCQ

1. Choose the correct alternative for each of the following : 1 each

(1) $\underline{\int_a^b} f dx =$ ....

(*a*) $\inf\{U(p, f) : p$ is a partition of $[a, b]\}$

(*b*) $\sup\{L(p, f) : p$ is a partition of $[a, b]\}$

(*c*) $\inf\{L(p, f) : p$ is a partition of $[a, b]\}$

(*d*) $\sup\{U(p, f) : p$ is a partition of $[a, b]\}$

P.T.O.

(2) If p^* is a refinement of p of $[a, b]$, then for a bounded function f .... .

(a) $L(p^*, f) \geq L(p, f)$ (b) $L(p^*, f) \leq L(p, f)$

(c) $L(p, f) \geq L(p^*, f)$ (d) $L(p, f) \leq L(p^*, f)$

(3) If f_1 and f_2 are integrable, then which of the following statements is correct ?

(a) $f_1 \pm f_2$ is integrable on $[a, b]$

(b) $f_1 f_2$ is integrable on $[a, b]$

(c) $|f|$ is integrable on $[a, b]$

(d) All of the above

(4) If a function f is bounded and integrable on each of the intervals $[a, c]$, $[c, b]$, $[a, b]$, where c is a point of $[a, b]$ then $\int_a^b f dx =$ ....

(a) $\int_a^b f dx + \int_a^b f dx$ (b) $\int_b^a f dx + \int_a^c f dx$

(c) $\int_a^c f dx + \int_c^b f dx$ (d) $\int_a^b f dx + \int_b^c f dx$

(5) A function f is bounded and integrable on $[a, b]$ and there exists a function F such that $F' = f$ on $[a, b]$, then $\int_a^b f dx =$ ....

(a) $F(b) - F(a)$ (b) $f(b) - f(a)$

(c) $F(a) - F(b)$ (d) $f(a) - f(b)$

(6) $\int_0^1 \frac{1}{\sqrt{x}} dx =$ ....

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

(7) The improper integral $\int_a^b f dx$ is said to be convergent at b, if :

(*a*) for every λ, $0 < \mu < b - a$, $\int_a^{b-\mu} f dx$ exists

(*b*) $\int_a^b f dx = \lim_{\mu \to 0^+} \int_a^{b-\mu} f dx$

(*c*) $\lim_{\mu \to 0^+} \int_a^{b-\mu} f dx$ exists and finite

(*d*) All of the above

(8) A series of the form

$$\frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

is called :

(*a*) Trigonometric series (*b*) Taylor's series

(*c*) Maclaurin's series (*d*) Power series

(9) A period function of bounded variation can be expressed as a ....

(*a*) Trigonometric series (*b*) Fourier series

(*c*) Taylor's series (*d*) Power series

(10) If f is an odd function, then a_n = ...

(*a*) $\frac{1}{\pi}\int_0^{\pi} f \cos nx \, dx$ (*b*) $\frac{1}{\pi}\int_{\pi}^{0} f \cos nx \, dx$

(*c*) $\frac{1}{\pi}\int_{-\pi}^{\pi} f \cos nx \, dx$ (*d*) $\int_{-\pi}^{\pi} f \cos nx \, dx$

2. Attempt any *two* of the following : 5 each

(*a*) Prove that a function f is integrable over $[a, b]$ iff there is a number I lying between $L(p, f)$ and $U(p, f)$ such that for any $\in > 0$, there exists a partition p of $[a, b]$ such that :

$$|U(p, f) - I| < \in \text{ and } |I - L(p, f)| < \in.$$

(*b*) If f is bounded and integrable on $[a, b]$, then prove that $|f|$ is also bounded and integrable on $[a, b]$. Moreover :

$$\left|\int_a^b f dx\right| \leq \int_a^b |f| \, dx.$$

(*c*) Show that the function f defined by :

$$f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$$

is not integrable on any interval.

3. Attempt any *two* of the following : 5 each

(*a*) If a function f is continuous on $[a, b]$, then prove that there exists a number ξ in $[a, b]$ such that :

$$\int_a^b f dx = f(\xi)(b-a).$$

(*b*) If f and g be two positive functions in $[a, b]$ such that :

$$\lim_{x \to a^+} \frac{f(x)}{g(x)} = l,$$

where l is a non-zero finite number, then prove that the two integrals

$$\int_a^b f dx \text{ and } \int_a^b g dx$$

converge and diverge together at a.

(*c*) Test the convergence of :

$$\int_0^{\pi/2} \frac{\sin x}{x^p} dx.$$

4. Attempt any *two* of the following : 5 each

(*a*) If f is bounded and integrable on $[-\pi, \pi]$ and if a_n, b_n are its Fourier coefficients, then prove that :

$$\sum_{n=1}^{\infty} \left(a_n^2 + b_n^2\right)$$

converges.

(*b*) If f is bounded and integrable in $[-\pi, \pi]$ and monotonic in $[-\delta, 0[$ and $]\,0, \delta]$, where $0 < \delta < \pi$, then prove that :

$$\frac{1}{2}a_0 = \sum_{n=1}^{\infty} a_n = \frac{f(0-)+f(0+)}{\pi}\int_0^{\infty} \frac{\sin x}{x}\,dx$$

where a_n, $n = 0, 1, 2, ...$ denote the Fourier's coefficients of f.

(*c*) Expand in a series of sines and cosines of multiple angles of x, the periodic function f with period 2π defined as

$$f(x) = \begin{cases} -1, & \text{for } -\pi < x < 0 \\ 1, & \text{for } 0 \le x \le \pi \end{cases}$$

Also calculate the sum of the series at $x = 0, \frac{\pi}{2}, \pm\pi$.