This question paper contains **10** printed pages]

BF—77—2016

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

(Old Course)

MATHEMATICS

Paper X (MT-205)

(Ring Theory)

(MCQ + Theory)

(Thursday, 20-10-2016) **Time : 2.00 p.m. to 3.00 p.m.**

Time—One Hour *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Each question carries **1** mark.

(*iii*) Negative marking system is applicable.

(*iv*) Choose one most correct answer from 4 alternatives.

MCQ

1. Let R be the set of integers mod 7 under addition and multiplication mod 7 i.e.

$$R = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}\}$$

then :

(A) $\overline{4}.\overline{5} = 6$ (B) $\overline{4}.\overline{3} = 7$

(C) $\overline{7} + \overline{7} = 1$ (D) $\overline{7}.\overline{7} = 1$

2. If R is the set of integers and + is the usual addition and • is the usual multiplication of integers, then :

(A) R is not ring

(B) R is a commutative ring with unit element

(C) R is a commutative ring but has no unit element

(D) R has no unit element

P.T.O.

3. If R is the set of integers mod 6, then :

(A) $\overline{2}\,.\,\overline{3} \neq \overline{0}$ (B) $\overline{2}\,.\,\overline{3} = \overline{6}$

(C) $\overline{2}\,.\,\overline{3} = \overline{0}$ (D) $\overline{2} + \overline{7} = \overline{0}$

4. If p is a prime number, then J_p, the ring of integers mod p is :

(A) integral domain but not field

(B) is a field

(C) has zero divisors

(D) none of the above

5. Let D be a finite integral domain. In order to prove that D is a field :

(A) We must produce an element $1 \in D$ such that $a.1 = a$ for every $a \in D$

(B) For every element $a \neq 0 \in D$. Produce an element $b \in D$ such that $ab = 1$

(C) Both (A) and (B)

(D) Only (B)

6. An integral domain D is said to be of characteristic 0 if the relation $ma = 0$ where $a \neq 0$ is in D and where m is an integer can hold only if :

(A) $m < 0$ (B) $m \geq 0$

(C) $m = 0$ (D) $m > 0$

7. If ϕ is a homomorphism of R into R' with kernel $I(\phi)$, then :

(A) $I(\phi)$ is a subgroup of R under addition

(B) If $a \in I(\phi)$ and $r \in R$, then both ar and ra are in $I(\phi)$

(C) Both (A) and (B)

(D) Only (B)

8. If U is an ideal of a ring R and $1 \in U$, then :

(A) $U \subset R$ but $R \not\subset U$ (B) $R \subset U$ but $U \not\subset R$

(C) $U \neq R$ (D) $U = R$

9. If U is an ideal of a ring R, then :

(A) $(U + a) + (U + b) = 2U + a + b$

(B) $(U + a)(U + b) = U + (a + b)$

(C) $(U + a) + (U + b) = U + (a + b)$

(D) $(U + a) + (U + b) = U + ab$

10. Let R be a commutative ring with unit element whose only ideals are (0) and R itself, then :

(A) R is a field

(B) R is only integral domain

(C) R is not integral domain

(D) R is not field

11. A ring R can be imbedded in a ring R' if :

(A) There is an isomorphism of R into R'

(B) There is an isomorphism of R into R

(C) There is an isomorphism of R' into R'

(D) None of the above

12. An integral domain is said to be a Euclidean ring if for every $a \neq 0$ in R there is defined a non-negative integer $d(a)$ such that :

(A) For all $a, b \in R$, both non-zero $d(a) \leq d(ab)$

(B) For any $a, b \in R$, both non-zero, there exists $t, r \in R$ such that $a = tb + r$, where either $r = 0$ or $d(r) < d(b)$

(C) Both (A) and (B)

(D) Only (A)

13. Consider two statements :

(I) Every Euclidean ring is a principal ideal ring

(II) Every principal ideal ring is a Euclidean ring

Then

(A) Both statements (I) and (II) are true

(B) Both statements (I) and (II) are false

(C) Only statement (I) is true

(D) Only statement (II) is true

14. If $a, b \in R$, then $d \in R$ is said to be a greatest common divisor of a and b :

(A) if $d | a$ and $d | b$

(B) whenever $c | a$ and c / b, then $c | d$

(C) Only (A) is true

(D) Both (A) and (B) are true

15. Which of the following is *false* ?

(A) If $a | b$ and $b | c$ then $a | c$

(B) If $a | b$ and $a | c$ then $a | b - c$

(C) If $a | b$ then $a | bx$ for all $x \in R$

(D) None of the above

16. Let R be a commutative ring with unit element. Two elements a and b in R are said to be associates if :

(A) $b = ua$ for all unit u in R

(B) $b = u + a$ for some unit u in R

(C) $b = ua$ for some unit u in R

(D) $b = u - a$ for some unit u in R

17. Let R be a Euclidean ring. Suppose that for $a, b, c \in R$, $a \mid bc$ but $(a, b) = 1$. Then :

(A) a is divisible by c (B) c is divisible by a

(C) a does not divide c (D) c divides a

18. The units in a commutative ring with a unit element :

(A) does not form a group

(B) forms group which is non-abelian

(C) forms abelian group

(D) none of the above

19. In a commutative ring with unit element the relation a is an associate of b is :

(A) not reflexive

(B) only transitive

(C) reflexive, symmetric and transitive

(D) only reflexive

20. Let p be a prime integer and suppose that for some integer c relatively prime to p. We can find integers x and y such that $x^2 + y^2 = cp$. Then p can be written as :

(A) The sum of squares of two integers

(B) The sum of two integers

(C) The difference of squares of two integers

(D) The product of squares of two integers

21. If p is a prime number of the form $4n + 1$, then we can solve the congruence :

(A) $x^2 \equiv 1 \bmod p$ (B) $x^2 \equiv -1 \bmod p$

(C) $x^2 \equiv 1 \bmod p^2$ (D) $x \equiv -1 \bmod p$

22. If $f(x)$, $g(x)$ are two non-zero elements of F[x], then $\deg(f(x)\, g(x))$:

(A) $\leq \deg f(x) + \deg g(x)$ (B) $\leq \deg f(x) . \deg g(x)$

(C) $\geq \deg f(x) + \deg g(x)$ (D) $= \deg f(x) + \deg g(x)$

23. Consider two statements :

(I) F[x] is a Euclidean ring

(II) F[x] is a Principal ideal ring

Then :

(A) Only statement (I) is true

(B) Only statement (II) is true

(C) Both statements (I) and (II) are true

(D) Both statements (I) and (II) are false

24. The property $a(b+c) = ab + ac$ for all $a, b, c \in R$ is known as :

(A) Commutative property (B) Associative property

(C) Idempotent law (D) Distributive law

25. If ϕ is a homomorphism of a ring R into R', then :

(A) $\phi(0) = 0$

(B) $\phi(-a) = -\phi(a)$ for every $a \in R$

(C) Both (A) and (B)

(D) Only (B) is true

26. If U, V are ideals of R, let :

$$U + V = \{u + U \mid u \in U, v \in V\}$$

Then :

(A) U + V is not subgroup of R under addition

(B) U + V is not ideal of R

(C) U + V is an ideal of R

(D) None of the above

27. Let R be a Euclidean ring and consider two statements :

(I) Any two elements a and b in R have a greatest common divisor d

(II) $d = \lambda a + \mu b$ for some $\lambda, \mu \in R$

Then :

(A) Both statements (I) and (II) are true

(B) Only statement (II) is false

(C) Only statement (I) is false

(D) Both statements (I) and (II) are false

28. Let R be a commutative ring with unit element. An element $a \in R$ is a unit in R if there exists an element $b \in R$ such that :

(A) $a + b = 1$

(B) $ab = 1$

(C) $ab = 0$

(D) $a - b = 1$

29. Let R be a Euclidean ring. Then every element in R :

(A) is a unit in R

(B) can be written as the product of a finite number of prime elements of R

(C) Both (A) and (B)

(D) Either (A) or (B)

30. If π is a prime element in the Euclidean ring R and π divides a_1, a_2, a_3 ...., a_n, then :

(A) π divides at least one $a_1, a_2, \ldots\ldots\ldots, a_n$

(B) π does not divide any of $a_1, a_2, \ldots\ldots\ldots, a_n$

(C) π divides at most one $a_1, a_2, \ldots\ldots\ldots, a_n$

(D) π divides at least two of $a_1, a_2, \ldots\ldots\ldots, a_n$

31. A necessary and sufficient condition that the element a in the Euclidean ring be a unit is that :

(A) $d(a) \neq d(1)$ (B) $d(a) = d(1)$

(C) $d(a) < d(1)$ (D) $d(a) \leq d(1)$

32. Let J[i] denote the set of all complex numbers of the form $a + ib$ where a and b are integers J[i] forms an integral domain which is called as :

(A) Domain of positive integers

(B) Domain of Gaussian integers

(C) Domain of integers

(D) Integral domain

33. F[x] is :

(A) commutative ring having no unit element

(B) commutative ring with unit element

(C) not integral domain

(D) not a Euclidean ring

34. Which of the following polynomials is irreducible over real field ?

(A) $x^2 - 1$ (B) $x^2 - 4$

(C) $x^2 - 16$ (D) $x^2 + 1$

35. Which of the following polynomials is irreducible over the rationals :

(A) $x^3 - 8$ (B) $x^3 - 9$

(C) $x^3 - 10$ (D) $x^3 - 11$

36. Let $f(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$ be a polynomial with integer coefficients. For which of the following conditions $f(x)$ is irreducible over the rationals :

(A) For some prime number p $p | a_n, p | a_1, p | a_2$ .... $p | a_0, p^2 | a_0$

(B) For some prime number p $p \nmid a_n, p | a_1, p | a_2$ .... $p | a_0, p^2 \nmid a_0$

(C) For some prime number p $p \nmid a_n, p | a_0, p | a_1, -p | a_2$ .... $p^2 | a_0$

(D) None of the above

37. The polynomial $f(x) = a_0 + a_1x + ... + a_nx^n$ where $a_0, a_1, ..., a_n$ are integers is said to be primitive if the greatest common divisor of $a_0, a_1, a_2, ..., a_n$ is :

(A) <1 (B) >1

(C) ≥ 1 (D) $=1$

38. The greatest common divisor of 25 and 31 is :

(A) 5 (B) 2

(C) 1 (D) 31

39. Consider two statements :

(I) Every integral domain is a field

(II) Euclidean ring possesses unit element.

Then :

(A) Both statements (I) and (II) are true

(B) Both statements (I) and (II) are false

(C) Only statement (I) is true

(D) Only statement (II) is true

40. If we consider R as the set of even integers, positive, negative and 0, + is usual addition and is the usual multiplication of integers, then :

(A) R does not satisfy all properties of ring

(B) R is a commutative ring without unit element

(C) R is a commutative ring with unit element

(D) For all $a, b \in R \quad a.b \neq b.a$