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BF—78—2016

FACULTY OF ARTS/SCIENCES

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2016

(Revised Course)

MATHEMATICS

Paper X

(Ring Theory)

(MCQ + Theory)

(Thursday, 20-10-2016) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First **30** minutes for Q. No. **1** and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball point pen to darken the circle on OMR sheet for Q. No. **1**.

(*v*) Negative marking system is applicable for Q. No. 1 (MCQ).

MCQ

1. Choose the *correct* alternative for each of the following : 10

(*i*) If R is the set of rational numbers under the usual addition and multiplication of rational numbers, then :

(A) R is a commutative ring

(B) R has a unit element

(C) R is a field

(D) All of these are correct

(*ii*) A commutative ring is an integral domain if :

(A) It has zero divisors (B) It has no zero divisors

(C) It has a unit element (D) None of these

P.T.O.

(*iii*) Let R and R' be two arbitray rings and define $\phi : R \to R'$ as $\phi(a) = 0$ for all $a \in R$, then :

(A) ϕ is not a homomorphism

(B) $I(\phi) = 0$

(C) ϕ is a homomorphism with $I(\phi) = R$

(D) ϕ is a homomorphism with $I(\phi) = 0$

(*iv*) If U is an ideal of R and $1 \in U$, then :

(A) $U = 0$ (B) $U = R$

(C) $U \neq R$ (D) All of these

(*v*) An ideal $M \neq R$ in a ring R is said to be a maximal ideal of R if whenever U is an ideal of R such that $M \subset U \subset R$, then :

(A) $R \neq U$ (B) $M \neq U$

(C) $R = U$ and $M = U$ (D) $R = U$ or $M = U$

(*vi*) In the Euclidean ring R, a and b in R are said to be relatively prime, if :

(A) their greatest common divisor is a unit of R

(B) a divides b

(C) b divides a

(D) their least common multiple is zero

(*vii*) If $p(x) = 1 + x - x^2$ and $q(x) = 2 + x^2 + x^3$, then $p(x) \cdot q(x) =$

(A) $2 + 2x + x^2 + 2x^3 + x^5$

(B) $3 + x + x^3$

(C) $2 + 2x - x^2 + 2x^3 - x^5$

(D) None of these

(*viii*) A polynomial $p(x)$ in $f(x)$ is said to be irreducible over F if whenever $p(x) = a(x).\ b(x)$ with $a(x), b(x) \in F(x)$, then :

(A) both $a(x), b(x)$ must have degree 0

(B) one of $a(x)$ or $b(x)$ has degree 0

(C) neither $a(x)$ nor $b(x)$ has degree 0

(D) both $a(x), b(x)$ has degree 1

(*ix*) If $f(x), g(x)$ are non-zero elements in $F(x)$, then :

(A) $\deg f(x) \geq \deg f(x).\ g(x)$

(B) $\deg f(x) \leq \deg g(x)$

(C) $\deg f(x) \leq \deg f(x).\ g(x)$

(D) $\deg f(x) = \deg f(x) + \deg g(x)$

(*x*) If $a \neq 0$, $b \neq 0$ are in R and $a \mid b$, $b \mid c$, then :

(A) $a \mid c$ (B) $c \mid a$

(C) $a = c$ (D) $a = bc$

Theory

2. Attempt any *two* of the following : 5 each

(*a*) If R is a ring, then for all $a, b \in$ R, prove that :

(*i*) $a.0 = 0.a = 0$

(*ii*) $a(-b) = (-a)b = -(ab)$

(*b*) If ϕ is a homomorphism of R in R', then prove that :

(*i*) $\phi(0) = 0$

(*ii*) $\phi(-a) = -\phi(a)$ for every $a \in$ R

(*c*) If $R = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}\}$ is the set of integers mod 7 under the addition and multiplipication mod 7, then find :

(*i*) $\bar{2} + \bar{3}$

(*ii*) $\bar{4} + \bar{5}$

(*iii*) $\bar{0} + \bar{6}$

(*iv*) $\bar{2} \cdot \bar{5}$

(*v*) $\bar{4} \cdot \bar{3}$

3. Attempt any *two* of the following : 5 each

(*a*) If U is an ideal of R, then prove that R/U is a ring.

(*b*) Prove that a Euclidean ring posseses a unit element.

(*c*) If U, V are ideals of R, let $U + V = \{u + v \mid u \in U, v \in V\}$, then prove that U + V is also an ideal.

4. Attempt any *two* of the following : 5 each

(*a*) If p is a prime number of the form $4n + 1$, then prove that we can solve the congruence $x^2 \equiv -1 \pmod{p}$.

(*b*) If $f(x)$, $g(x)$ are two non-zero elements of F(x), then prove that : $\deg(f(x).\, g(x)) = \deg f(x) + \deg g(x)$.

(*c*) Prove that $x^2 + 1$ is irreducible over the integers mod 7.