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R—66—2017

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

MARCH/APRIL, 2017

(Revised Course)

MATHEMATICS

Paper IX

(Real Analysis—II)

(MCQ+Theory)

(Friday, 31-3-2017) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes are for Question No. **1** (MCQ) and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball point pen to darken the circle of correct choice in OMR answer-sheet.

(*v*) Negative marking system is applicable for MCQs.

(MCQs)

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) If f is bounded and integrable on $[a, b]$, then there exists a number λ lying between the bounds of f such that :

$\int_a^b f\, dx$ is equal to :

(*a*) $b - a$ (*b*) $a - b$

(*c*) $\lambda(b - a)$ (*d*) $\lambda(a - b)$

P.T.O.

(*ii*) If f is a bounded function on $[a, b]$, then to every $\in > 0$, there corresponds $\delta > 0$ such that :

(*a*) $U(P, f) > \int_{a}^{-b} f\ dx + \in$ (*b*) $U(P, f) < \int_{a}^{-b} f\ dx + \in$

(*c*) $L(P, f) < \int_{-a}^{b} f\ dx - \in$ (*d*) $L(P, f) < \int_{-a}^{b} f\ dx + \in$

(*iii*) If f is Riemann integrable on $[a, b]$, then :

(*a*) $\left|\int_{a}^{b} f\ dx\right| \le \int_{a}^{b} |f|\ dx$

(*b*) $\left|\int_{a}^{b} f\ dx\right| \ge \int_{a}^{b} |f|\ dx$

(*c*) $\left|\int_{a}^{b} f\ dx\right| = \int_{a}^{b} |f|\ dx$

(*d*) None of the above

(*iv*) $\int_{1}^{2} f\ dx$, where $f = 3x + 1$, is equal to :

(*a*) $\frac{2}{11}$ (*b*) $\frac{3}{2}$

(*c*) $\frac{1}{2}$ (*d*) $\frac{11}{2}$

(*v*) Let f is a non-negative continuous function on $[a, b]$ and $\int_{a}^{b} f(x) dx = 0$, then $f(x)$ is equal to :

(*a*) 1 (*b*) 0

(*c*) –1 (*d*) ± 1

(*vi*) If a function f is continuous on $[a, b]$, then there exists a number ξ in $[a, b]$ such that $\int_a^b f\, dx$ is equal to :

(*a*) $b - a$

(*b*) $a - b$

(*c*) $f(\xi)(b - a)$

(*d*) $f(\xi)(a - b)$

(*vii*) The improper integral $\int_a^b \frac{dx}{(x-a)^n}$ conveges if and only if :

(*a*) $n < 1$

(*b*) $n > 1$

(*c*) $n \geq 1$

(*d*) $n \leq 1$

(*viii*) For a periodic function of period 2π, then $\int_{-\pi}^{\pi} f\, dx$ is equal to :

(*a*) $\int_{\alpha}^{2\pi} f\, dx$

(*b*) $\int_{\alpha}^{\pi} f\, dx$

(*c*) $\int_{\alpha}^{\alpha+\pi} f\, dx$

(*d*) $\int_{\alpha}^{\alpha+2\pi} f\, dx$

(*ix*) If a function f is bounded and integrable in $[0, a]$, $a > 0$, and monotone in $]0, \delta]$, $0 < \delta < a$, then $\lim_{n\to\infty} \int_0^a f \frac{\sin nx}{x} dx$ is equal to :

(*a*) $f(0^-) \int_0^{\infty} \frac{\sin x}{x} dx$

(*b*) $f(0^+) \int_0^{\infty} \frac{\sin x}{x} dx$

(*c*) $f(0) \int_{\infty}^{0} \frac{\sin x}{x} dx$

(*d*) $f(0^+) \int_{-\infty}^{\infty} \frac{\sin x}{x} dx$

P.T.O.

(x) If f is an even function then a_n is equal to :

(a) $\frac{1}{\pi}\int_{0}^{\pi} f \cos nx \, dx$ (b) $\frac{1}{\pi}\int_{-\pi}^{\pi} f \cos nx \, dx$

(c) $\frac{2}{\pi}\int_{-\pi}^{\pi} f \cos nx \, dx$ (d) $\frac{2}{\pi}\int_{0}^{\pi} f \cos nx \, dx$

(Theory)

2. Attempt any *two* of the following : 5 each

(a) For any *two* partitions P_1, P_2 prove that :

$$L(P_1, f) \leq U(P_2, f).$$

(b) Prove that the oscillation of a bounded function f on an interval $[a, b]$ is the supremum of the set $\{|f(x_1) - f(x_2)| : x_1, x_2 \in [a, b]\}$ of numbers.

(c) Show that x^2 is integrable on any interval $[0, k]$.

3. Attempt any *two* of the following : 5 each

(a) If a function f is bounded and integrable on $[a, b]$, then prove that the function F defined as :

$$F(x) = \int_{a}^{x} f(t)\,dt, \; a \leq x \leq b$$

is continuous on $[a, b]$.

(b) If f and g are integrable on $[a, b]$ and g keeps the same sign over $[a, b]$, then prove that there exists a number μ lying between the bounds of f such that :

$$\int_{a}^{b} fg \, dx = \mu \int_{a}^{b} g \, dx.$$

(c) Examine the convergence of :

$$\int_{0}^{2} \frac{dx}{(2x - x^2)}.$$

4. Attempt any *two* of the following : 5 each

(*a*) If f is bounded and integrable in $[-\pi, \pi]$ and monotonic in $[-\delta, 0[$ and $]0, \delta]$, where $0 < \delta < \pi$, then :

$$\frac{1}{2}a_0 = \sum_{n=1}^{\infty} a_n = \frac{f(0-) + f(0+)}{\pi} \int_0^{\infty} \frac{\sin x}{x}\, dx$$

where a_n, $n = 0, 1, 2, \ldots\ldots\ldots\ldots$ denote the Fourier's coefficients of f.

(*b*) For a periodic function of period 2π, prove that :

$$\int_{\alpha}^{\beta} f\, dx = \int_{\alpha+2\pi}^{\beta+2\pi} f\, dx$$

α, β, γ being any numbers whatsoever.

(*c*) Find the Fourier series of the periodic function f with period 2π, defined as :

$$f(x) = \begin{cases} 0, & \text{for } -\pi < x \leq 0 \\ x, & \text{for } 0 \leq x \leq \pi \end{cases}$$

What is the sum of the series at $x = 0$?