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R—79—2017

FACULTY OF SCIENCE

B.Sc. (Second Year) (Fourth Semester) EXAMINATION

MARCH/APRIL, 2017

(Revised Course)

MATHEMATICS

Paper X

(Ring Theory)

(MCQ+Theory)

(Monday, 3-4-2017) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes for Question No. **1** and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball point pen to darken the circle on OMR-sheet for Q. No. **1**.

(*v*) Negative marking system is applicable for Q. No. **1** (MCQs).

(MCQs)

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) R is the set of integers, positive, negative and 0; + is usual addition and '.' the usual multiplication of integers, then :

(*a*) R is a commutative ring

(*b*) R is a commutative ring with unit element

(*c*) R is a commutative ring but has no unit element

(*d*) None of the above

P.T.O.

(*ii*) An integral domain D is said to be of finite characteristic if there exists a positive integer m such that ....

(*a*) $ma = 0$ for all $a \in D$

(*b*) $ma \neq 0$ for all $a \in D$

(*c*) $m \neq 0$

(*d*) $a = 0$

(*iii*) Let R be a ring, $R' = R$ and define $\phi(x) = x$ for every $x \in R$, then :

(*a*) ϕ is not a homomorphism

(*b*) $I(\phi) = R$

(*c*) ϕ is a homomorphism and $I(\phi) = R$

(*d*) ϕ is a homomorphism and $I(\phi)$ consists only of 0

(*iv*) A non-empty subset U of R is said to be a ideal of R if :

(*a*) U is a subgroup of R under addition

(*b*) For every $u \in U$ and $r \in R$, both ur and ru are in U

(*c*) Both (*a*) and (*b*)

(*d*) None of the above

(*v*) Which of the following is/are true ?

(*a*) If a/b and b/c then a/c

(*b*) If a/b and a/c then $a/(b \pm c)$

(*c*) If a/b and a/bx for all $x \in R$

(*d*) All of the above

(*vi*) Let R be a commutative ring with unit element. Two elements a and b in R are said to be associates if :

(*a*) $b = ua$ for any u in R

(*b*) $b = ua$ for some unit u in R

(*c*) $b \neq ua$ for u in R

(*d*) $b = a$

(*vii*) The polynomial $x^2 + 1$ is ....

(*a*) reducible over the real field

(*b*) irreducible over the complex field

(*c*) irreducible over the complex field but not over the real field

(*d*) irreducible over the real field but not over the complex field

(*viii*) The content of the polynomial $f(x) = a_0 + a_1 x + ... + a_n x^n$, where the a's are integers, is ....

(*a*) the greatest common divisor of the integers $a_0, a_1, a_2, ..., a_n$

(*b*) the greatest common divisor of the integers $a_1, a_2 ...., a_n$

(*c*) the greatest common divisor of the integers $a_2, a_3 ...., a_n$

(*d*) the greatest common divisor of the integers $a_3, a_4, ..., a_n$

(*ix*) Which of the following is/are *true* ?

(*a*) F[x] is an integral domain

(*b*) F[x] is an Euclidean ring

(*c*) F[x] is a principal ideal ring

(*d*) All of the above

(*x*) If $p(x) = 1 + x - x^2$ and $q(x) = 2 + x^2 + x^3$, then $p(x) . q(x) =$ ....

(*a*) $2 + x^4 + 3x^5$ (*b*) $2 + 2x - x^2 + 2x^3 - x^5$

(*c*) $2 + 3x^3 - 4x^5$ (*d*) $2 + x^5 + x^4 + x^3 + 7x^2$

(Theory)

2. Attempt any *two* of the following : 5 each

(*a*) If ϕ is a homomorphism of R into R′ with kernel I(ϕ), then prove that :

(*i*) I(ϕ) is a subgroup of R under addition.

(*ii*) If $a \in$ I (ϕ) and $r \in$ R then both ar and ra are in I(ϕ).

(*b*) If R is a ring, then for all $a, b \in$ R prove that :

(*i*) $a(-b) = (-a)b = -(ab)$

(*ii*) $(-a)(-b) = ab$.

P.T.O.

(*c*) If $R = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}\ \overline{4}, \overline{5}\}$ is the set of integers mod 6 under addition and multiplication then find :

(*i*) $\overline{3} + \overline{4}$

(*ii*) $\overline{2} + \overline{5}$

(*iii*) $\overline{1} + \overline{5}$

(*iv*) $\overline{2} . \overline{4}$

(*v*) $\overline{3} . \overline{5}$

3. Attempt any *two* of the following : 5 each

(*a*) Let R be a Euclidean ring and let A be an ideal of R. Then prove that there eixsts an element $a_0 . \in$ A such that A consists exactly of all a_0x as x ranges over R.

(*b*) Let R be an integral domain with unit element and supose that for $a, b \in$ R both a/b and b/a are true. Then prove that $a = ub$, where u is a unit in R.

(*c*) Prove that if $[a, b] = [a', b']$ and $[c, d] = [c', d']$ then prove that :

$$[a, b]\ [c, d] = [a', b']\ [c', d'].$$

4. Attempt any *two* of the following : 5 each

(*a*) If $f(x)$ and $g(x)$ are primitive polynomials, then prove that $f(x)\ g(x)$ is a primitive polynomial.

(*b*) Prove that if R is an integral domain, then so is $R[x]$

(*c*) Prove that $x^2 + x + 1$ is irreducible over F, the field of integers mod 2.