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R—92—2017

FACULTY OF SCIENCE

B.Sc. (Second Year) (Fourth Semester) EXAMINATION

MARCH/APRIL, 2017

MATHEMATICS

Paper XI

(Partial Differential Equations)

(MCQ & Theory)

(Thursday, 6-4-2017) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Use only black point pen for first question.

(*iii*) Darken only *one* circle for most correct answer of each MCQ.

(*iv*) Negative marking system is applicable for first question.

MCQ

1. Choose the most *correct* answer of the following (attempt *all*) : 1 each

(*i*) Partial differential coefficient ..... is denoted by '*t*'.

(*a*) $\frac{\partial^2 z}{\partial x \partial y}$ (*b*) $\frac{\partial^2 z}{\partial x^2}$

(*c*) $\frac{\partial^2 z}{\partial y^2}$ (*d*) $\frac{\partial z}{\partial x}$

(*ii*) Partial differential equation is formed by eliminating arbitrary ..... .

(*a*) Constants (*b*) Variables

(*c*) Constants or variables (*d*) None of these

P.T.O.

(*iii*) The order of partial differential equation is ..... as that of the order of highest differential coefficient in it.

(*a*) Greater than (*b*) Same

(*c*) Less than (*d*) None of these

(*iv*) With usual notations the form of Lagrange's linear partial differential equation is ..... .

(*a*) $P_p + Q_q < R$ (*b*) $P_p + Q_q = R$

(*c*) $P_p + Q_q > R$ (*d*) None of these

(*v*) An equation of the type

$$a_0 \frac{\partial^n z}{\partial x^n} + a_1 \frac{\partial^n z}{\partial x^{n-1} \partial y} + \ldots\ldots\ldots + a_n \frac{\partial^n z}{\partial y^n} = F(x, y)$$

is called ..... .

(*a*) Homogeneous linear partial differential equation

(*b*) *n*th order partial differential equation

(*c*) Partial differential equation with constant coefficients

(*d*) All of the above

(*vi*) The auxiliary equation of :

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$$

is ..... .

(*a*) $m^2 - 4m + 4 = 0$ (*b*) $m^2 + 4 = 0$

(*c*) $m - 4 = 0$ (*d*) $m^2 + 4m - 4 = 0$

(*vii*) With usual notations the equations

$$Rdpdy + Tdqdx - Vdxdy = 0$$

and

$$Rdy^2 - Sdxdy + Tdx^2 = 0$$

are called ..... equations.

(*a*) Lagrange's (*b*) Charpit's

(*c*) Monge's (*d*) None of these

(*viii*) The partial differential equations :

$$-\frac{\partial V}{\partial x} = L\frac{\partial I}{\partial t}, -\frac{\partial I}{\partial x} = C\frac{\partial V}{\partial t}$$

are called ..... equations.

(*a*) Radio (*b*) Wave

(*c*) Laplace (*d*) None of these

(*ix*) Which of the following is Laplace equation in two dimensions ?

(*a*) $\frac{\partial u}{\partial v} = a^2 \frac{\partial^2 u}{\partial x^2}$ (*b*) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = a$

(*c*) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial t^2}$ (*d*) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(*x*) The solution of wave equation is obtained using ..... method.

(*a*) Charpit's (*b*) D'Almbert's

(*c*) Lagrange's (*d*) None of these

Theory

2. Attempt any *two* of the following : 5 each

(*a*) Form a partial differential equation from :

$$x^2 + y^2 + (z - c)^2 = a^2.$$

(*b*) Solve :

$$\frac{\partial^2 z}{\partial x^2 \partial y} = \cos(2x + 3y).$$

(*c*) Solve the following PDE :

$$yq - xp = z$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

P.T.O.

3. Attempt any *two* of the following : 5 each

(*a*) Explain the rule for finding the complementary function of the linear homogeneous PDE of *n*th order with constant coefficients.

(*b*) Solve :

$$p^2 + q^2 = 1$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

(*c*) Find the general integral of the equation :

$$\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y.$$

4. Attempt any *two* of the following : 5 each

(*a*) Solve the wave equation :

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

by D'Almbert's method.

(*b*) Obtain the solution of the wave equation :

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

using method of separation of variables.

(*c*) Find the solution of :

$$\frac{\partial^2 u}{\partial x^2} = h^2 \frac{\partial y}{\partial t}$$

for which $u(0, t) = 0, = u(l, t)$, $u(x, 0) = \sin\frac{\pi x}{l}$ by method of variables separable.