This question paper contains **6** printed pages]

V—63—2017

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2017

(Revised Course)

MATHEMATICS

Paper II

(Real Analysis)

(MCQ + Theory)

(Friday, 10-11-2017) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes are for Q. No. 1 (MCQ) and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball pen to darken circle of correct choice in OMR answer sheet.

(*v*) Negative marking system is applicable for MCQ.

MCQ

1. Choose the *correct* alternative for each of the following : 1 each

(1) A necessary and sufficient condition for the integrability of a bounded function f is that to every $\in > 0$, there corresponds $\delta > 0$ such that for every partition p of $[a, b]$ with norm $\mu(p) < \delta$ such that :

(A) $U(p, f) - L(p, f) \leq \in$

(B) $U(p, f) - L(p, f) < \in$

(C) $U(p, f) - L(p, f) > \in$

(D) $U(p, f) - L(p, f) \geq \in$

P.T.O.

(2) If f is bounded and integrable on $[a, b]$, then $|f|$ is :

(A) Bounded on $[a, b]$

(B) Integrable on $[a, b]$

(C) Both (A) and (B)

(D) Only (A)

(3) The Riemann sum of f over $[a, b]$ relative to partition p is given by :

(A) $s(p,f) = \sum_{i=1}^{n} m_i \Delta x_i$ (B) $s(p,f) = \sum_{i=1}^{n} m_i \Delta x_i$

(C) $s(p,f) = \sum_{i=1}^{n} f(t_i) \Delta x_i$ (D) $s(p,f) = \sum_{i=1}^{n} m_i m_i \Delta x_i$

(4) $\int_{-1}^{1} |x| \, dx$ is equal to :

(A) ±1 (B) –1

(C) 0 (D) 1

(5) The value of $\int_0^3 [x] \, dx$ is equal to :

(A) 3 (B) –3

(C) 0 (D) ∞

(6) The improper integral

$$\int_a^b \frac{dx}{(x-a)^n}$$

converges if and only if :

(A) $n > 1$ (B) $n < 1$

(C) $n = 0$ (D) $n = 1$

(7) The improper integral $\int_a^b f\, dx$ converges at a if and only if to every $\in > 0$ there corresponds $\delta > 0$ such that :

(A) $\left|\int_{a+\lambda_1}^{a+\lambda_2} f\, dx\right| < \in,\ 0 < \lambda_1, \lambda_2 < \delta$

(B) $\int_{a+\lambda_1}^{a+\lambda_2} f\, dx < \in,\ 0 < \lambda_1, \lambda_2 < \delta$

(C) $\int_{a+\lambda_1}^{a+\lambda_2} f\, dx \le \in,\ 0 < \lambda_1, \lambda_2 < \delta$

(D) $\left|\int_{a+\lambda_1}^{a+\lambda_2} f\, dx\right| \le \in,\ 0 < \lambda_1, \lambda_2 < \delta$

(8) For a periodic function of period 2π, then $\int_{-\pi}^{\pi} f(x)\, dx$ is equal to :

(A) $\int_{-\pi}^{\pi} f(r+x)\, dx$, r being any number

(B) $\int_{-\pi}^{\pi} f(r-x)\, dx$, r being any number

(C) $\int_{-\pi}^{\pi} f(r)$, r being any number

(D) None of the above

(9) The Fourier series of the periodic function f with period 2π, defined as :

$$f(x) = \begin{cases} 0, & \text{for} \quad -\pi < x \le 0 \\ x, & \text{for} \quad 0 \le x \le \pi \end{cases}$$

the sum of series at $x = 0$ is :

(A) π (B) π^2

(C) $\pi^2/8$ (D) $\pi^2/4$

P.T.O.

(10) If a function f is bounded integrable and piecewise monotonic in $[0, \pi]$, then the sum of the sine series $\sum b_n \sin nx$, where b_n is equal to :

(A) $\int_0^{\pi} \sin nx \, dx$

(B) $-\int_0^{\pi} \sin nx \, dx$

(C) $\frac{1}{\pi}\int_0^{\pi} \sin nx \, dx$

(D) $\frac{2}{\pi}\int_0^{\pi} f \sin nx \, dx$

Theory

2. Attempt any *two* of the following : 5 each

(*a*) If f is a bounded function on $[a, b]$, then to every $\in > 0$, there corresponds $\delta > 0$ such that :

(*i*) $U(p,f) < \int_a^{-b} f \, dx + \in$

(*ii*) $L(p,f) > \int_a^{-b} f \, dx - \in$

for every partition p of $[a, b]$ with norm $\mu(p) < \delta$.

(*b*) If a function f is bounded and integrable on each of the intervals $[a, c]$, $[c, b]$, $[a, b]$, where c is a point of $[a, b]$ then prove that :

$$\int_a^b f \, dx = \int_a^c f \, dx + \int_c^b f \, dx.$$

(*c*) Show that the function f defined as $f(x) = \frac{1}{2^n}$ when :

$$\frac{1}{2^{n+1}} < x \leq \frac{1}{2^n}, \quad (n = 0, 1, 2 \,.......)$$

is integrable on $[0, 1]$.

3. Attempt any *two* of the following : 5 each

(*a*) If f and g are integrable on $[a, b]$ and g keeps the same sign over $[a, b]$ then there exists a number μ lying between the bouds of f such that :

$$\int_a^b fg\,dx = \mu \int_a^b g\,dx.$$

(*b*) If f and g are two positive functions in $[a, b]$ such that :

$$\lim_{x \to a+o} \frac{f(x)}{g(x)} = l,$$

where l is a non-zero finite number, then the two integrals $\int_a^b f\,dx$ and $\int_a^b g\,dx$ converge and diverge together $a + a$.

(*c*) Test the convergence of :

$$\int_0^1 \frac{dx}{\sqrt{1 - x^3}}.$$

4. Attempt any *two* of the following : 5 each

(*a*) If f is bounded and integrable on $[-\pi, \pi]$ and if a_n, b_n are its Fourier coefficients, then :

$$\sum_{n=1}^{\infty} \left(a_n^2 + b_n^2\right),$$

converges.

P.T.O.

(*b*) If a function ϕ is bounded and integrable on the interval $[a, b]$ then as $n \to \infty$

$$A_n = \int_a^b \phi \cos nx \, dx \to 0 \text{ and}$$

$$B_n = \int_a^b \phi \sin nx \, dx \to 0 .$$

(*c*) Find the Fourier series consisting of sine terms only, which represents the periodic function :

$$f(x) = x \text{ in } 0 \le x \le \pi .$$