This question paper contains **4** printed pages]

V—75—2017

FACULTY OF ARTS/SCIENCE

B.Sc. (Second Year) (Fourth Semester) EXAMINATION

NOVEMBER/DECEMBER, 2017

(Revised Course)

MATHEMATICS

Paper—X

(Ring Theory)

(MCQ+Theory)

(Monday, 13-11-2017) **Time : 2.00 p.m. to 4.00 p.m.**

Time— Two Hours *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes are for Q. No. 1 (MCQ) and remaining time for other questions.

(*iii*) Use black ball pen to darken circle of correct choice in OMR answer sheet.

(*iv*) Figures to the right indicate full marks.

(*v*) Negative marking scheme is applicable for MCQ.

(MCQ)

1. Choose the *correct* alternative for each of the following : 1 each

(*i*) R is the set of even integers under the usual operations of addition and multiplication, then R is :

(*a*) a non-commutative ring

(*b*) a commutative ring without unit element

(*c*) a commutative ring with unit element

(*d*) not a ring

P.T.O.

(*ii*) A ring is said to be a division ring, if its :

(*a*) non-zero elements form a group under multiplication

(*b*) non-zero elements form a group under addition

(*c*) non-zero elements doesn't form a group under multiplication

(*d*) None of the above

(*iii*) If n objects are distributed over m places and if $n > m$, then :

(*a*) All places receives exactly one object

(*b*) All places receives atmost one object

(*c*) Some places receives at least two objects

(*d*) Some places receives no object

(*iv*) If U, V are ideals of R, and $\text{U} + \text{V} = \{u + v \mid u \in \text{U}, v \in \text{V}\}$, then :

(*a*) U + V is not a subgroup of R under addition

(*b*) U + V is not an ideal

(*c*) For any $r \in \text{R}$, $(u + v)r \notin \text{U} + \text{V}$

(*d*) U + V is an ideal

(*v*) An ideal $\text{M} \neq \text{R}$ in a ring R is said to be a maximall ideal of R if whenever U is an ideal of R such that $\text{M} \subset \text{U} \subset \text{R}$, then :

(*a*) $\text{R} \neq \text{U}$ and $\text{M} \neq \text{U}$

(*b*) either R = U or M = U

(*c*) R = U but $\text{M} \neq \text{U}$

(*d*) None of the above

(*vi*) A ring R′ is called an over-ring of R, if :

(*a*) R can not be imbedded in R′

(*b*) R′ can be imbedded in R

(*c*) R can be imbedded in R′

(*d*) None of the above

(*vii*) If $a, b \in$ R, then $d \in$ R is said to be a greatest common divisor of a and b, if :

(*a*) d/a and d/b

(*b*) whenever c/a and c/b then c/d

(*c*) both (*a*) and (*b*) must be true

(*d*) (*a*) is true but (*b*) is not true

(*viii*) If π is a prime element in the Euclidean ring R and π/ab where $a, b \in$ R then :

(*a*) π divides at lesat one of a or b

(*b*) π neither divides a nor b

(*c*) π must divide both a and b

(*d*) a, b both divides π

(*ix*) Which of the following polynomial is primitive polynomial ?

(*a*) $2x^2 + 4x + 6$ (*b*) $3x^3 + 9$

(*c*) $x^3 + 2x^2 + 3x + 5$ (*d*) $5x^2 + 10x + 5$

(*x*) Which of the following polynomial is irreducible over the field of real numbers ?

(*a*) $x^2 - 4$ (*b*) $x^2 + 5x + 6$

(*c*) $x^2 - 25$ (*d*) $x^2 + 1$

2. Attempt any *two* of the following : 5 each

(*a*) Define Integral domain and prove that a finite integral domain is a field.

(*b*) Prove that the homomorphism ϕ of R into R′ is an isomorphism if and only if I(ϕ) = 0.

(*c*) Let R be a ring, R′ = R and define $\phi(x) = x$ for every $x \in$ R, then show that ϕ is a homomorphism and hence find kernel of ϕ.

3. Attempt any *two* of the following : 5 each

(*a*) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

(*b*) Let R be a Euclidean ring and $a, b \in$ R. If $b \neq 0$ is not a unit in R, then prove that $d(a) < d(ab)$.

(*c*) If U is an ideal of R and I $\in$ U, then prove that U = R.

4. Attempt any *two* of the following : 5 each

(*a*) If $f(x)$ and $g(x)$ are primitive polynomials then prove that $f(x)\, g(x)$ is also a primitive polynomial.

(*b*) If p is a prime number of the form $4n + 1$, then $p = a^2 + b^2$ for some integers a, b.

(*c*) Show that $x^2 + x + 1$ is irreducible over F, then field of integers mod 2.