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AO—62—2018

FACULTY OF ARTS/SCIENCE

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

MARCH/APRIL, 2018

(CBCS/CGPA Pattern)

MATHEMATICS

Paper IX

(Real Analysis–II)

(MCQ + Theory)

(Saturday, 24-3-2018) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes are for Q. No. **1** (MCQ) and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball pen to darken circle of correct choice in OMR answer sheet.

(*v*) Negative marking system is applicable for MCQ.

MCQ

1. Choose the correct alternative for each of the following : 1 each

(1) U(n, f) – L(p, f) is called :

(A) An upper sum (B) A lower sum

(C) An oscillatory sum (D) None of these

(2) If p_1 and p_2 are two partitions of $[a, b]$ such that $p_1 \geq p_2$, then :

(A) p_1 refines p_2

(B) p_1 is finer than p_2

(C) p_1 is a refinement of p_2

(D) All of the above

P.T.O.

(3) For any two partitions p_1, p_2 :

(A) $L(p_1, f) \leq L(p_2, f)$ (B) $L(p_1, f) \leq U(p_2, f)$

(C) $U(p_1, f) \leq U(p_2, f)$ (D) None of these

(4) A function f is integrable over $[a, b]$ iff for $\in > 0$, there exists $\delta > 0$ such that if p, p' are any *two* partitions of $[a, b]$ with mesh less than δ, then :

(A) $|S(p, f) - S(p', f)| < \in$ (B) $|S(p, f) - S(p', f)| > \in$

(C) $|S(p, f) + S(p', f)| < \in$ (D) $|S(p, f) + S(p', f)| > \in$

(5) $\int_a^b f'(x)\, dx =$

(A) $f(b) = f(a)$ (B) $f(a) - f(b)$

(C) $f(b) - f(a)$ (D) None of these

(6) If a function f is continuous on $[a, b]$, then there exists a number Σ_1 in $[a, b]$ such that $\int_a^b f dx =$

(A) $f(\Sigma_1)(a - b)$ (B) $f(\Sigma_1)(b - a)$

(C) $(b - a)$ (D) $(a - b)$

(7) $\int_a^b f dx$ is called an improper integral if ....

(A) f has singular points in $[a, b]$

(B) Limits of integration a or b or both infinite

(C) Either (A) or (B)

(D) Neither (A) nor (B)

(8) $\int_0^b f \dfrac{\sin nx}{\sin x} dx$ is called ....

(A) Dirichlet's integral (B) Double integral

(C) Triple integral (D) Integral

(9) If f is an even function, $f(-x) = f(x)$, for all x, then $f\cos nx$ is an even function is ....

(A) $\int_0^{\pi} f\cos nx\,dx$ (B) $\frac{2}{\pi}\int_0^{\pi} f\cos nx\,dx$

(C) $\frac{1}{\pi}\int_0^{\pi} f\cos nx\,dx$ (D) $\frac{-1}{\pi}\int_0^{\pi} f\cos nx\,dx$

(10) If the function is .... on the interval $[-\pi, \pi]$, then also its Fourier coefficients approach zero as $n \to \infty$.

(A) Continuous (B) Discontinuous

(C) Piecewise continuous (D) None of these

Theory

2. Attempt any *two* of the following : 5 each

(*a*) Prove that A bounded function f is integrable on $[a, b]$ iff for every $\in > 0$ there exists a partition p of $[a, b]$ such that :

$$U(p, f) - L(p, f) < \in.$$

(*b*) Prove that every continuous function is integrable.

(*c*) Show that $(3x + 1)$ is integrable on $[1, 2]$ and

$$\int_1^2 (3x+1)\,dx = \frac{11}{2}.$$

3. Attempt any *two* of the following : 5 each

(*a*) A function f is bounded and integrable on $[a, b]$ and there exists a function F such that $F' = f$ on $[a, b]$. Then prove that :

$$\int_a^b f dx = F(b) - F(a).$$

P.T.O.

(*b*) Prove that the improper integral $\int_a^b \frac{dx}{(x-a)^n}$ converges if and only if $n < 1$.

(*c*) Test the convergence of $\int_0^1 \frac{dx}{\sqrt{1-x^3}}$.

4. Attempt any *two* of the following : 5 each

(*a*) If f is bounded and integrable on $[-\pi, \pi]$ and if an, bn are its Fourier coefficients, then prove that :

$$\sum_{n=1}^{\infty} (a_n^2 + b_n^2) \text{ converges.}$$

(*b*) For a periodic function of period 2π, prove that :

(*i*) $$\int_{-\pi}^{\pi} f dx = \int_{\alpha}^{\alpha+2\pi} f dx$$

(*ii*) $$\int_{-\pi}^{\pi} f(x)\, dx = \int_{-\pi}^{\pi} f(\gamma + x)\, dx$$

where α, γ being any numbers.

(*c*) Find the Fourier series consisting of sine terms only, which represents the periodic function :

$$f(x) = x \text{ in } 0 \le x \le \pi .$$