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AO—74—2018

FACULTY OF ARTS AND SCIENCE

B.A./B.Sc. (Second Year) (Fourth Semester) EXAMINATION

MARCH/APRIL, 2018

(CBCS/CGPA)

MATHEMATICS

Paper X

(Ring Theory)

(MCQ & Theory)

(Tuesday, 27-03-2018) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) First 30 minutes for Question No. **1** and remaining time for other questions.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use black ball point pen to darken the circle on OMR sheet for Question No. **1**.

(*v*) Negative marking system is applicable for Q. No. **1**.

MCQ

1. Choose the *correct* alternative for each of the following : 1

(*i*) If R is the set of even integers under the usual operations of addition and multiplication, then :

(*a*) R is not a ring

(*b*) R is a ring but not commutative

(*c*) R is a commutative ring but has no unit element

(*d*) R is a commutative ring with unit element

(*ii*) If R is a commutative ring, then $a \neq 0 \in$ R is said to be a zero-divisor if there exists a, $b \in$ R, $b \neq 0$ such that :

(*a*) $ab = 1$ (*b*) $ab \neq 0$

(*c*) $ab < 0$ (*d*) $ab = 0$

P.T.O.

(*iii*) A mapping ϕ from the ring R into the ring R′ is said to be a homomorphism if :

(*a*) $\phi(a + b) = \phi(a)\ \phi(b)$ (*b*) $\phi(a + b) = \phi(a) + \phi(b)$

(*c*) $\phi(ab) = \phi(a)\ \phi(b)$ (*d*) Both (*b*) and (*c*) is true

(*iv*) If U is an ideal of R and $1 \in$ U, then :

(*a*) U $\subseteq$ R only (*b*) R $\subseteq$ U only

(*c*) U is not subset of R (*d*) U = R

(*v*) Consider two statements :

(I) Every integral domain is a field.

(II) Every field is integral domain. Then :

(*a*) Only statement (II) is true

(*b*) Only statement (I) is true

(*c*) Both statements (I) and (II) are true

(*d*) Both statements (I) and (II) are false

(*vi*) If U is an ideal of the ring R, then :

(*a*) R/U may or may not be ring

(*b*) R/U is a ring but not homomorphic image of R

(*c*) R/U is a ring and it is a homomorphic image of R

(*d*) None of the above

(*vii*) If $a, b \in$ R, then $d \in$ R is said to be a greatest common divisor of a and b if :

(*a*) d / a and d / b

(*b*) Wherever c / a and c / b then c / d

(*c*) Both (*a*) and (*b*) must true

(*d*) Both (*a*) and (*b*) need not be true

(*viii*) If p is a prime number of the form $4n + 1$, then we can solve the congruence :

(*a*) $x^2 \equiv 1 \pmod{p}$ (*b*) $x^2 \equiv -1 \pmod{p}$

(*c*) $x \equiv -1 \pmod{p}$ (*d*) $x \equiv 1 \pmod{p}$

(*ix*) If the primitive polynomial $f(x)$ can be factored as the product of two polynomials having rational coefficients then it can be factored as :

(*a*) The product of the two polynomials having integer coefficients

(*b*) The sum of two polynomials having rational coefficients

(*c*) The sum of two polynomials having real coefficients

(*d*) None of the above

(*x*) Which of the following is a primitive polynomial ?

(*a*) $x^3 + x + 1$ (*b*) $2x^2 + 4x$

(*c*) $3x^2 + 9x$ (*d*) $8x^2 + 16x$

Theory

2. Attempt any *two* of the following : 5 each

(*a*) Prove that a finite integral domain is a field.

(*b*) Let $J(\sqrt{2})$ be all real numbers of the form $m + n\sqrt{2}$ where m, n are integers, then prove that $J[\sqrt{2}]$ forms a ring under usual addition and multiplication of real numbers.

(*c*) If ϕ is a homomorphism of R into R′ with kernel $I(\phi)$, then prove that :

(*i*) $I(\phi)$ is a subgroup of R under addition.

(*ii*) If $a \in I(\phi)$ and $r \in R$, then both ar and ra are in $I(\phi)$.

3. Attempt any *two* of the following : 5 each

(*a*) Let R be a commutative ring with unit element whose only ideas are (0) and R itself. Then prove that R is a field.

(*b*) Prove that a Euclidean ring possesses a unit element.

(*c*) Prove that a necessary and sufficient condition that the element a in the Euclidean ring be a unit is that $d(a) = d(1)$.

4. Attempt any *two* of the following : 5 each

(*a*) Let p be a prime integer and suppose that for some integer c relatively prime to p we can find integers x and y such that $x^2 + y^2 = cp$. Then prove that p can be written as the sum of squares of two integers.

(*b*) Prove that if $f(x)$ and $g(x)$ are primitive polynomials, then $f(x)\, g(x)$ is a primitive polynomial.

(*c*) Show that $x^2 + x + 1$ is irreducible over F where F is the field of integers mod 2.