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W—100—2018

FACULTY OF SCIENCE

B.Sc. (Second Year) (Fourth Semester) EXAMINATION

OCTOBER/NOVEMBER, 2018

(CBCS/CGPA Pattern)

MATHEMATICS

Paper XI

(Partial Differential Equations)

(MCQ & Theory)

(Monday, 22-10-2018) **Time : 2.00 p.m. to 4.00 p.m.**

Time—2 *Hours* *Maximum Marks*—40

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Use only black point pen for first question.

(*iii*) Darken only one circle for most correct answer of each MCQ.

(*iv*) Negative marking system is applicable for first question.

MCQ

1. Choose the most *correct* answer of the following : 10

(*i*) In partial differential equation the symbol 't' is used to denote :

(*a*) $\frac{\partial z}{\partial x}$ (*b*) $\frac{\partial z}{\partial y}$

(*c*) $\frac{\partial^2 z}{\partial x^2}$ (*d*) $\frac{\partial^2 z}{\partial y^2}$

(*ii*) Identify the Lagrange's linear equation from the list given below :

(*a*) $Pp + Qq = R$ (*b*) $Rr + Ss + Tt = V$

(*c*) $P + Q = R$ (*d*) $R + S + T = V$

P.T.O.

(*iii*) The auxiliary equations of $yq - xp = z$ are :

(*a*) $\frac{dx}{-1} = \frac{dy}{1} = \frac{dz}{z}$ (*b*) $\frac{dx}{-x} = \frac{dy}{y} = \frac{dz}{z}$

(*c*) $\frac{dx}{y} = \frac{dy}{z} = \frac{dz}{x}$ (*d*) $\frac{dx}{-z} = \frac{dy}{x} = \frac{dz}{y}$

(*iv*) The solution of partial differential equation $p^2 + q^2 = 1$ is :

(*a*) $z = \sqrt{1 - a^2}\,y + c$ (*b*) $z = x + \sqrt{1 - a}\,y + c$

(*c*) $z = ax + \sqrt{1 - a^2}\,y + c$ (*d*) $z = ay + \sqrt{x^2 - 1}$

(*v*) The solution of the equation $z = px + qy + f(p, q)$ is :

(*a*) $z = ax + by + f(x, y)$ (*b*) $z = ax + by$

(*c*) $z = f(a, b)$ (*d*) $z = ax + by + f(a, b)$

(*vi*) In the equation $(a_0 D^2 + a_1 DD' + a_2 D'^2)z = 0$ the auxiliary equation is :

(*a*) $a_0 m^2 + a_1 m + a_2 = 0$ (*b*) $a_0 m + a_1 m^2 + a_2 = 0$

(*c*) $a_0 + a_1 + a_2 = 0$ (*d*) $m + m^2 + 1 = 0$

(*vii*) In partial differential equation $f(D, D')z = F(x, y)$. Its particular integral, when $F(x, y) = e^{ax + by}$ is :

(*a*) $\frac{e}{f(a, b)}$ (*b*) $\frac{1}{f(a, b)}$

(*c*) $\frac{ax + by}{f(a, b)}$ (*d*) $\frac{e^{ax + by}}{f(a, b)}$

(*viii*) The equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is called :

(*a*) Laplace equation in two-dimension

(*b*) Two-dimensional heat flow

(*c*) Both (*a*) and (*b*)

(*d*) None of the above

(*ix*) In the steady state, $u(x, y)$ does not change with time t, then which of the following is true ?

(*a*) $\frac{\partial u}{\partial t} = t$ (*b*) $\frac{\partial u}{\partial t} = 0$

(*c*) $\frac{\partial x}{\partial t} = 1$ (*d*) $\frac{\partial u}{\partial x} = 0$

(*x*) Identify the Laplace equation in polar co-ordinates :

(*a*) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$ (*b*) $\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial u}{\partial \theta} = 0$

(*c*) $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r^2}\frac{\partial u}{\partial \theta} + \frac{\partial^2 u}{\partial \theta^2} = 0$ (*d*) $\frac{\partial^2 u}{\partial r^2} - \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$

Theory

2. Attempt any *two* of the following : 5 each

(*a*) Explain the working rule for finding solution of partial differential equation :

$$P_p + Q_q = R$$

with usual notations.

(*b*) Solve :

$$\frac{\partial^3 z}{\partial x^2 \cdot \partial y} = \cos (2x + 3y).$$

P.T.O.

(*c*) Solve :

$$(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx.$$

3. Attempt any *two* of the following : 5 each

(*a*) Explain the Monge's method to solve the equation :

$$R_r + S_s + T_t = V$$

where R, S, T, V are functions of x, y, z, p and q.

(*b*) Solve :

$$px + qy = pq.$$

(*c*) Solve :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x.\partial y} - 6\frac{\partial^2 z}{\partial y^2} = x + y.$$

4. Attempt any *two* of the following : 5 each

(*a*) Discuss the two-dimensional heat flow to get Laplace equation.

(*b*) Solve :

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

by method of separation of variables.

(*c*) Solve the wave equation :

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}$$

under the condition :

$u = 0$ when $x = 0$ and $x = \pi$

$\frac{\partial u}{\partial t} = 0$ when $t = 0$ and $u(x, 0) = x$, $0 < x < \pi$.